THE UNIVERSITY OF ALBERTA

COMPARISON OF TWO PROBLEM SOLVING APPROACHES IN GRADE EIGHT MATHEMATICS

by

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The undersigned certify that they have read, and recommended to the Faculty of Graduate Studies for acceptance, a thesis entitled "Comparison of Two Problem Solving Approaches in Grade Eight Mathematics," submitted by Ralph Anton Gorrie in partial fulfilment of the requirements for the degree of Master of Education.

ABSTRACT

The purpose of this study was to exhibit a reasonable and unsophisticated but careful method of gathering experimental data which might be adopted by a typical rural school jurisdiction as a basis for evaluation of the relative effectiveness of two problem-solving approaches at the grade eight level. One group of 128 students studied a traditional approach using a traditional textbook, Winston Mathematics in grade seven and eight. The other group of 139 students studied a modified Winston Mathematics program designed by the investigator. The experimental modified program was based on Seeing Through Arithmetic in grade six and emphasized the Gestalt-ratio approach to problem-solving.

In June, 1963 the traditional class at the end of grade eight wrote the Otis Test of Mental Ability, Beta, Form EM and a special Problem Solving Eight test designed by the investigator from problem types found in Winston Mathematics. The following June, 1964, the experimental group wrote the same two tests in its grade eight year. Each class then wrote the grade nine departmental examination in mathematics after studying Mathematics for Canadians, Book 1 in the usual manner.

It was found that on the Problem Solving Eight test

at the end of grade eight, there was no significant difference between the group mean scores obtained by the two treatment groups. There was no significant difference found between the means of the two treatment groups in either the rate or non-rate problem subtests.

It was found that the Gestalt-ratio approach to problem-solving provided students with a significantly faster method of preparing correct written solutions to problems.

On the grade nine mathematics departmental examination the traditional students performed significantly better than the modern students who employed the Gestalt-ratio approach to problem-solving. The average ability student scored significantly higher as an ability group studying the traditional materials.

The use of analysis of variance as a statistical technique accompanied by appropriate profiles appeared to be a useful means of providing for unsophisticated educational research for the small rural school jurisdiction in Alberta.

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CHAPTER I

THE PROBLEM AND DEFINITIONS OF TERMS USED

Problem-solving pervades all curricula. Methodology of and a new emphasis on problem-solving is becoming the educator's concern in curriculum design. In particular, the mathematics curriculum in Alberta has received considerable attention and consequent revision since the early 1960's. A major re-emphasis on problem-solving technique resulted in a new look at this most crucial area of mathematics. Has this new look actually enhanced the pupil's ability to solve problems?

I. THE PROBLEM

Statement of the Problem

It was the main purpose of this study to compare two methods of teaching word problem-solving techniques in junior high school mathematics, utilizing a careful, but unsophisticated research methodology.

Importance of the Study

A view generally held by a majority of mathematics teachers with whom the investigator has been associated is that one of the predominant problem instructional areas in mathematics curriculum is that of producing successful problem-solvers in our school system. The significance of suc-

cessful problem-solvers is expressed by Bingham when she writes

> Since man's progress is measured in proportion to his ability to solve problems, and since today's children are tomorrow's adult problem-solvers, teachers need to be concerned with the growth manifested in problem-solving facility by the children under their direction.[1]

It logically follows that concern for investigating problem-solving approaches and their relative effectiveness is easily justified. The more recent investigations of problem-solving approaches and programs by Lindstedt,[2] Harrison,[3] and Worbets[4] revealed some evidence of a degree of superiority of problem-solving ability depending upon the method of instruction.

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[1]Alma Bingham, "Improving Children's Facility in Problem-Solving," (New York: Bureau of Publications, Teachers College, Columbia University, 1958), p. 2.

[2]Sidney A. Lindstedt, "Changes in Patterns of Thinking Produced by A Specific Problem Solving Approach in Elementary Arithmetic" (unpublished Dissertation, University of Wisconson, 1962)

[3]Donald B. Harrison, "An Analysis of The Effectiveness of Three Mathematics Programs at The Grade Eight Level" (unpublished Master's thesis, The University of Alberta, 1964)

[4]William T. Worbets, "Comparison of Problem Solving Proficiency of Grade Nine Students in Four Different Mathematics Programs" (unpublished Master's thesis, The University of Alberta, 1966)

From these investigations it became somewhat evident that the more rigorous methods of symbolic language associated with the modern approach to problem-solving were apparently not as effective as its authors claimed. Yet the publishers of the Seeing Through Arithmetic and Seeing Through Mathematics programs, Gage and Company,[5] make claim of the advantages of having children use equations to solve arithmetic "story problems". They claim:

> In almost any real-life problem situation that is at all complicated, we find it helpful to make an outline of the essential details before we try to decide what to do about our problem. Yet the traditional textbook suggests no way for children to organize facts of an arithmetic problem situation, or to write them down. In effect children are expected to keep everything in mind and figure out a solution all at one time.[6]

The Alberta Junior High School Mathematics Subcommittee in its guide for Junior High School Mathematics curriculum emphasizes the need for the development of systematic methods of analysing problems and of presenting their solutions.[7] Prior to this statement the subcommittee showed favour to an approach which stressed:

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[5]W.J. Gage Limited, When Parents Ask About Arithmetic Today, (Lithographed, 1959), p. 10.

[6]Gage, loc. cit.

[7]Alberta Junior High School Mathematics Subcomittee, Curriculum Guide Grade VII Mathematics, Interim, September, 1965. p. 1.

(a) the statement of the problem situation in the form of a mathematical statement followed by computations and the interpretation relating the answer to the original situation,

(b) the use of a ratio or ratio-pair approach to all problems to which it can be applied.[8]

As a result of this new omphasis, several school systems in September, 1961 were authorized to experiment with two series of "modern" mathematics at the elementary school level. The County of Beacver introduced Seeing Through Arithmetic[9] and Arithmetic We Need[10] that year. Overwhelming teacher acceptance of Seeing Through Arithmetic resulted in the desire of only one text series in 1962. In the short space of one year teachers were convinced of the apparent superiority of the new textbook series over the out-dated Study Arithmetic.[11] It was their conviction that problem-solving ability was enhanced by this approach.

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[8] Alberta Junior High School Mathematics Subcommittee, Junior High School Mathematics Bulletin, April 1963 (mimeographed)

[9] Henry Van Engen and others, Seeing Through Arithmetic, Books 3 to 6, (Toronto: W.J. Gage Limited, 1959)

[10] Guy T Buswell and others, Arithmetic We Need, Books 3 to 6, (Toronto: Ginn and Company, 1959)

[11] J.W. Studebaker and others, Study Arithmetic, Books 3 to 6, (Toronto: W.J. Gage Limited, 1949)

With this obvious emphasis on experimental programs in all aspects of curriculum design and the numerous experimental textbook authorizations, rural school jurisdictions were professionally responsible to continue to forge ahead in educational endeavors. However, inherent in any program of endeavor is evaluation. One of the most important roles of evaluation in the classroom is the improvement of our instruction.[12] Through evaluation one is able to determine the effectiveness of the techniques, materials, and the content of our teaching. Thus evaluation gives school authorities a basis for building the curriculum and the selecting of methods of teaching that will develop a desired mathematical competence.

Consequently, the necessity of an unsophisticated treatment of data must be realized since neither the sample or the data would appropriately lend themselves to more rigid statistical manipulation and inference.

Hypotheses

For the purpose of this study, students studying traditional materials, Study Arithmetic and Winston Mathematics[13]

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[12] Donovan A. Johnson, Introduction, Evaluation In Mathematics, (The National Council of Teachers of Mathematics, 26th. Yearbook, 1961) p. 3.

[13] H.L. Stein and others, Winston Mathematics, Intermediate 1 and 2, (Toronto: Holt Rinehart and Winston of Canada, Limited, 1954)

are referred to as (TT) students; and students studying the modern mathematics, Seeing Through Arithmetic or Arithmetic We Need and the modified program in grades seven and eight are referred to as (MT) students.

The null hypotheses tested were as follows:

I. On the PS8 * test there is no significant difference between the group mean scores attained by the (TT) students and the (MT) students.

II. On the PS8 test (rate section) there is no significant difference between the group mean scores attained by the (TT) students and the (MT) students.

III. On the PS8 test (non-rate section) there is no significant difference between the group mean scores attained by the (TT) students and the (MT) students.

IV. On the PS8 test there is no significant difference in mean times required by the (TT) students and the (MT) students.

V. On the M9 ** test there is no significant difference between the mean stanine scores attained by the (TT) students and the (MT) students.

Descriptive statistics with appropriate tables of means and profiles were employed to better picture the apparent effects of the two treatments in the PS8 multiple-step type problem-solving, and PS8 single-step type problem-solving. Also described were the effects of treatments in relation to PS8 completion time by achievement groups;

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* This abbreviation, PS8, will refer to the Problem Solving Eight test.

** This abbreviation, M9, will refer to the Mathematics Nine Departmental Examination.

PS8 scores by sex; and PS8 scores by time level groups.

Delimitations

The investigator recognizes the many factors which may affect the proficiency of students in problem-solving activities. This study is only concerned with ability as measured by an I.Q. test and the effect of two approaches to problem-solving on a special problem-solving test designed by the investigator. Though the Winston Mathematics, Intermediate 1 and 2 were the texts for the experimental group as well as the control group, the uncontaminated effect of the investigator's teaching guide is unknown. Further limitations must be realized because of the unknown problem-solving proficiency each group had attained prior to the grade six year. For the study it is assumed that each treatment group was of equal proficiency at the commencement of the study. No pre-test was administered because of administrative circumstances. Since each class used Mathematics for Canadians, Book 1,[14] a traditional text, the effect of forgetting since the grade eight year was not considered. Though teachers in each grade remained unchanged, the effect of the teacher characteristics as a variable over the period of three years would place further

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[14]Henry Bowers and others, Mathematics for Canadians, Book 1, (J.M. Dent and Sons, Canada, Limited and The Macmillan Company of Canada Limited, 1947)

limitations on the study. The measuring instrument, PS8, for problem-solving may have or may not have been appropriate. Finally, the investigator fully realizes the limitation of the study is affected by the design of the research. Due to circumstances at the time the study was conceived, the design could not readily satisfy the assumption of randomness; and further, the time lapse nature of the sample would impose a further restriction. Thus, within the scope of these limitations, the study tried to answer questions previously posed.

II. DEFINITION OF TERMS AND ABBREVIATIONS

Definition of Terms

Modified Program. The program specially prepared by the investigator to provide continuity in the Junior High School mathematics program until new authorizations were approved by the Alberta Junior High School Mathematics Subcommittee.

Multiple-step Problem. The problem type which in order to be solved must be completed in at least two separate operations in which information obtained by employing one binary operation is required to obtain the necessary placeholder in a second operation. For example, PS8 test item 6:

> For spending money, George received 75¢ per week and Alice 50¢ per week. What was the total for six weeks for both children? [15]

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[15] Appendix A

Two binary operations are required to solve this problem no matter how the problem is attacked. Either the operation of multiplication or addition must be executed first.

Non-rate Problem. The type of problem in which the situation is either additive or subtractive in nature and can not be solved using a ratio equation.

Problem-solving. The activity required by a student to solve a verbal problem of the textbook variety according to a predetermined method of attack learned by the student.

Rate Problem. The type of problem situation which is multiplicative or divisive in nature and is easily solved using a ratio equation.

Single-step Problem. The problem type which requires but one binary operation to solve the problem. For example, PS8 test item 5:

> When Mr. Peters had driven 250 miles, he had used 13 gallons of gas. To the nearest whole mile, how many miles was this per gallon?[16]

Note that this problem type requires the one operation of division to obtain the answer.

Stanine. The statistical distribution employed by the Examinations Branch of the Department of Education of Alberta according to the normal curve of grade nine results

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[16] Appendix A

and as described by Ferguson.[17]

Abbreviations.

<u>APS</u>. Average problem solvers who attained a score between 9 and 17(inclusive) on the PS8 test.

<u>AV</u>. Average ability group who attained a score between 96 and 105(inclusive) on the <u>OTIS</u>.

<u>AW</u>. The group of students designated average workers who required between 80 and 102(inclusive) minutes to complete the PS8 test.

<u>FW</u>. The group of students designated fast workers who required 79 minutes or less to complete the PS8 test.

<u>GPS</u>. Good problem solvers who attained a score between 18 and 27(inclusive) on the PS8 test.

<u>HA</u>. Above average ability group who attained a score between 106 and 115(inclusive) on the <u>OTIS</u>.

<u>HI</u>. High ability group who attained a score of 116 or better on the <u>OTIS</u>.

<u>LA</u>. Low average ability group who attained a score between 86 and 95(inclusive) on the <u>OTIS</u>.

<u>LO</u>. Low ability group who attained a score of 85 or less on the <u>OTIS</u>.

<u>M9</u>. The grade nine departmental examination in

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[17]George A. Ferguson, <u>Statistical Analysis in Psychology and Education</u>, (New York: McGraw-Hill Book Company, Inc., 1959). p. 223

mathematics

MT. The group of students who studied Study Arithmetic, Books 3, 4, and 5, Seeing Through Arithmetic, Book 6 and the modified Winston Mathematics, Intermediate 1 and 2. This group maintained problem-solving skills taught in Seeing Through Arithmetic, Book 6.

OTIS. The ability test, Otis Quick-Scoring Mental Ability Tests: New Edition, Beta EM,[18] administered at the end of grade eight to each group in the study.

PPS. Poor problem solvers who attained a score between 0 and 8(inclusive) on the PS8 test.

PS8. The special problem-solving test designed by the investigator and administered at the end of grade eight to each group in the study, and the corresponding raw scores.[19]

SCAT. The Cooperative School and College Ability Test administered to the grade nine students of the province each June.

SW. The group of students designated slow workers who required 103 minutes or more to complete the PS8 test.

TT. The group of students who studied the Study

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[18] Arthur S. Otis, Otis Quick-Scoring Mental Ability Tests: New Edition, Beta EM,(New York: Harcourt, Brace and World, Inc., 1954).

[19] Appendix A

Arithmetic series in elementary school and *Winston Mathematics* series in grades seven and eight and designated the control group in the study.

III. OUTLINE OF THE REPORT

The present chapter is an introduction and a preview of the research study. A review of the literature pertinent to the problem is contained in Chapter II. Chapter III outlines the experimental design and statistical procedures employed in the study. The results of the investigation are detailed in Chapter IV. Chapter V concludes the report with a summary, conclusions, limitations, and implications for further research and study.

CHAPTER II

REVIEW OF THE LITERATURE

Problem-solving in every field of endeavour has received a good share of concern in the literature. In particular, the field of mathematics has endeavoured to research the intricacies of the methods of problem-solving employed by the individual. Lazerte[1] as early as 1933, employing the "envelope test" attempted to establish patterns of thinking of elementary school children. The literature is virtually replete with learned papers on problem-solving. Particular emphasis is found in the area of problem-solving processes, the multifactors which affect problem-solving facility and the myriad suggestions for attempting to improve problem-solving facility of students. Only until recently has research on comparison of problem-solving methods received considerable attention. This later development in Alberta, particularly, is likely related to the new mathematics textbook authorizations of the Department of Education.

Consequently, this chapter will be devoted to the following topics: (1) a review of recent investigations of

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[1] M.E. Lazerte, The Development of Problem Solving Ability in Arithmetic, (Toronto: Clarke Irwin and Company, 1933), pp. 5-21.

problem-solving approaches; (2) an analysis of the problem-solving process; (3) a discussion of the traditional textbook approach to problem-solving; and (4) a discussion of the modern textbook approach to problem-solving.

REVIEW OF RECENT INVESTIGATIONS

Three of the more recent local studies concerned with effectiveness of various problem-solving approaches provide a reference for this study. Each study, Lindstedt,[2] Harrison,[3] and Worbets[4] sampled Alberta students in relation to different teaching approaches to problem-solving.

Lindstedt's[5] study related to elementary arithmetic. He sought to identify patterns of thinking related to the Seeing Through Arithmetic approach that were different from

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[2]Sidney A. Lindstedt, "Changes in Patterns of Thinking Produced by A Specific Problem Solving Approach in Elementary Arithmetic," (unpublished Dissertation, University of Wisconson, 1962.)

[3]Donald B. Harrison, "An Analysis of The Effectiveness of Three Mathematics Programs at The Grade Eight Level,"(unpublished Master's thesis, University of Alberta, 1964.)

[4]William T. Worbets, "Comparison of Problem Solving Proficiency of Grade Nine Students in Four Different Mathematics Programs," (unpublished Master's thesis, University of Alberta, 1966.)

[5]Lindstedt, *loc. cit.*

those related to the traditional approach. Findings indicated fundamentally that the experimental group using Seeing Through Arithmetic materials related the mathematical model more closely to the action of the problem. Further, the experimental group showed superiority in solving problems with imaginative settings and with unfamiliar word and number symbols. His further analysis indicated, however, no statistically significant difference between the two groups in problem-solving competence, when the competence is measured by a test that contains only the basic form of each problem type.

Harrison[6] researched the effectiveness of three mathematics programs at the grade eight level. Basically again, the relative effectiveness of the new approaches and the conventional or traditional approach were compared. Harrison compared grade eight classes who studied Seeing Through Mathematics, Exploring Modern Mathematics, and Winston Mathematics on the Iowa Tests of Basic Skills (grade eight arithmetic concepts and problem solving sections) as well as a Special Mathematics Understanding test. Results indicated the Exploring Modern Mathematics students scored significantly higher on the Special Mathematical Understandings test than the Seeing Through Mathematics students did, who in turn scored significantly higher than the Winston

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[6]Harrison, op. cit., pp. 136 - 141.

Mathematics students on the same test. Further, on the *Iowa Problem Solving* test, which was largely computationally oriented, the scores obtained by the *Exploring Modern Mathematics* students and the *Winston Mathematics* did not differ significantly. He felt that a peculiar phenomenon occurred when the conventional method produced better results than the modern method on the *Iowa Problem Solving* test.

Worbets[7] continued the study to grade nine in a similar design employing the *Iowa Problem Solving* subtest. The conventional group studied *Mathematics for Canadians, Bk. 1* in grade nine. The investigation concluded that on the *Iowa Problem Solving* test there were no significant differences among the group mean scores of the students studying the four different mathematics programs. Further however, on the *Special Problem Solving* test at the end of grade nine the *Exploring Modern Mathematics* students and the *Seeing Through Mathematics-Exploring Modern Mathematics* students scored significantly higher than either the *Seeing Through Mathematics* or *Mathematics for Canadians* students. Worbets further found that on the *Special Problem Solving* test at the end of grade nine, the low ability students studying *Exploring Modern Mathematics* textbooks achieved significantly higher than the students studying the three other mathematics programs.

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[7]Worbets, *op. cit.*, pp. 96 - 101.

ANALYSES OF PROBLEM SOLVING

What is problem-solving? For that matter, what is a problem? The dictionary[8] defines a problem as a question; or even a difficult question, but really something to be worked out, such as a problem in arithmetic. Apparently the statement must be interrogative and must have some degree of difficulty to be categorized as a problem.

Bingham writes:

> What is a problem? A problem is a hindrance that blocks an individual's presently constituted powers to achieve a desirable goal. A problem exists for an individual when he experiences obstacles in attempting to attain a particular objective or understanding.[9]

Polya[10] suggests that problem-solving actually means finding a way out of a difficulty, a way around an immediate obstacle, attaining an aim which was originally not immediately attainable. Cohen and Johnson[11] state a good

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[8]"Problem", Thorndike-Barnhart Desk Dictionary(1951) p. 619.

[9]Alma Bingham, Improving Children's Facility in Problem Solving (New York: Bureau of Publications, Teachers College, Columbia University, 1958), p. 7.

[10]George Polya, Mathematics Discovery Volume I, (New York: John Wiley and Sons, Inc., 1962), p. v.

[11]Louis S. Cohen and David C. Johnson, "Some Thoughts about Problem Solving," The Arithmetic Teacher, 14: 261-62, April, 1967, p. 261.

problem in mathematics is one which can be thought of as a new situation for the individual who is called upon to solve it. In particular Cohen and Johnson write:

> The novel situation is such that the path to the goal(the solution) is blocked and the individual's fixed patterns of behavior or habitual responses are not sufficient for removing the block. Hence deliberation must take place.[12]

Obviously, then, problem-solving is synonymous with mental activity. Lindstedt[13] emphatically states that problem-solving involves thinking and in reality if we are to teach a child to solve an arithmetic problem we are to teach the child to think. Lindstedt continues by noting:

> This is because every problem is unique. If it were not unique it would not be a problem -- it might be an exercise, repititious in design and skill producing during its execution, but unless it has some new and different relationships and involvements, it is not a problem.[14]

Polya[15] would support this contention. He suggests that solving problems is a specific achievement of the specific gift of man - intelligence. There is, consequently, apparently complete agreement on problem-solving as a mental activity of the individual.

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[12] Ibid.

[13] S.A. Lindstedt, "The Problem of Solving Arithmetic Problems," (W.J. Gage Limited, lithographed), p. 15.

[14] Ibid.

[15] Polya, op. cit., p. 118.

Manheim[16] attempted to classify word problems into two categories: real and imaginary word problems. He suggested that a real word problem is one which employs English or other non-mathematical language to ask a question about a physical phenomenon; the laws, of course, governing the particular phenomenon being presupposed. His second category includes imaginary word problems which differ from real word problems in actually one respect; the laws so governing, which relate quantifiable words are explicitly enunciated. A word problem is synonymous with verbal problem. According to Glennon[17] a verbal problem is a problem of the usual textbook variety.

Accepting the fact that a problem, hence a verbal problem, is an individual matter according to the degree of blockage existing between the individual and the solution, it would follow that to one individual a proposed question may not be a problem to the same extent as to another individual. Consequently, individual differences become a pertinent factor in the process of problem-solving.

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[16] Jerome Manheim, "Word Problems or Problems with Words," The Mathematics Teacher, 54: 234, April, 1961.

[17] Vincent J. Glennon, What Does Research Say about Arithmetic, (Washington: Association for Supervision and Curriculum Development, A department of the National Education Association, 1958).

A problem may exist for some but not for all. Succinctly put by Henderson and Pingry; "What is one student's problem is another student's exercise, and a third student's frustration."[18]

The various expressions of thought in the literature about the nature of problem-solving may suggest that it is more meaningful to think of problem-solving as a complex of many functions rather than as some single unitary function. This is the contention of Gross and McDonald.[19]

According to O'Brien[20] the essential elements of a problem are (1) there is something you want, and (2) you do not know how to get at it. It is generally accepted, as Lindstedt[21] states and O'Brien reaffirms that inherent in every problem is some degree of difficulty, and consequently,

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[18] Kenneth B. Henderson and Robert E. Pingry, "Problem Solving in Mathematics, Chapter VIII", The Learning of Mathematics, Its Theory and Practice, Howard Fehr, Editor, pp. 228 - 69, (National Council of Teachers of Mathematics, 21st. Yearbook, 1955), p. 232.

[19] Richard E. Gross and Frederick J. McDonald, "The Problem Solving Approach," Phi Delta Kappan, 39: 260, March, 1958.

[20] Katherine E. O'Brien, "Problem Solving," The Mathematics Teacher, 49: 2, p. 79, February, 1956.

[21] Lindstedt, loc. cit.

[22] O'Brien, loc. cit.

if there is a degree of difficulty, then a problem will generate mental activity. Butler and Wren[23] note that problem-solving situations require not only that the student be able to do the things that need to be done, but also make an important decision on what things must be done and in what order. One school of thought conceives problem-solving as a process consisting of three parts: (1) the ongoing sustaining activity of the individual, where the problem-solver must feel that the obtaining of the answer is his goal and only when the goal is attained will he find the problem satisfying; (2) the blocking of the behavior normally employed by the individual in obtaining his goal, thus finding a problem; and (3) the student beginning to think and to figure out ways of removing the block and thereby attaining his goal.[24]

It is becoming more apparent that solving problems involves more than the problem; it also involves the individual, probably to a greater extent than teachers realize. Fundamentally, then, one might suggest that the crucial factor may be the extent to which the student's ego becomes involved in the problem.[25]

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[23] Charles H. Butler and Lynwood Wren, _The Teaching of Secondary Mathematics_, (New York: McGraw-Hill Book Company, 1965), p. 273.

[24] Henderson and Pingry, _op. cit._, pp. 229-30

[25] _Ibid._, p. 232.

Henderson and Pingry interpret Dewey's analysis of the problem-solving process in this way:

> A problem situation is presented to an individual. The individual immediately develops or initiates some inhibition of direct action which results in a conscious awareness of a forked-road situation. Such a situation prepares for intellectualization of the felt difficulty which leads to definition of the problem.[26]

Apparently with the problem defined, the individual begins to identify various hypotheses which will direct observations and other operations in the collection of factual material. The individual is now in the position to elaborate on each of the hypotheses by reasoning about and testing of the hypotheses. He will then act on the basis of a particular hypothesis selected, thereby proceeding to the ultimate test.[27]

Gross and McDonald would support this view when from their review of the literature they advance the theory that the problem-solving process involves three essential functions: (1) an orientation function; (2) an elaborative function and analytic function; and (3) a critical function.[28]

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[26] Ibid., p. 236.

[27] loc. cit.

[28] Gross and McDonald, op. cit., pp. 260-62.

Further support for this view is documented by O'Brien[29] when she concedes that there is general agreement on these three steps in the process of problem-solving: (1) analyzing the hypothesis; (2) analyzing the conclusion; and(3) finding the connection between the hypothesis and the conclusion. One would have to assume, of course, that orientation of some manner would have to precede drafting of any kinds of hypotheses. O'Brien may be oversimplifying the situation when she very tersely puts the statement: "(1) What have you got? (2) What do you want? and (3) How can you use what you have to get what you want?"[30]

Interesting studies have been made to detect the possibility of general patterns of problem-solving thinking which may characterize sizable groups of individuals. Buswell[31] augmented such a study in 1956 with high school and college students. The evidence indicated that in the case of college students variations rather than uniformity was the major characteristic in problem-solving processes. He concluded that the thinking of the students in the study

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[29]O'Brien, op. cit., p. 84.

[30]Ibid.

[31]Guy T. Buswell and Kersh Y. Bert, "Patterns of Thinking in Solving Problems," University of California Publications in Education, Vol. 12, No. 2.(Berkley: University of California Press, 1956), p. 131.

indicated an absence of a generalized mode of problem-solving.[32]

If any pattern of problem-solving exists or should exist, likely the most all inclusive sequence of steps found in the literature which might prove acceptable is that formulated by Bingham:

> 1. identifying the problem and feeling the need to pursue it;
>
> 2. seeking to clarify the problem and understand its nature, scope and subproblems;
>
> 3. collecting data and information related to the problem;
>
> 4. selecting and organizing the data which apply most pertinently to the crux of the problem;
>
> 5. determining the various possible solutions in view of the assembled data and knowledge of the problem;
>
> 6. evaluating the solutions and selecting the best one for the situation;
>
> 7. putting the solution into action; and
>
> 8. evaluating the problem-solving process employed.[33]

In summary then, it is apparent that problem-solving technique has individual character, and no matter what pattern is taught in the classroom one really is making an attempt to teach the child to think.

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[32] _Ibid._, p. 139.

[33] Bingham, _op. cit._, p. 13.

THE TRADITIONAL PROBLEM SOLVING APPROACH

A review of textbooks of arithmetic of the traditional variety would seem to indicate an actual lack of any definite pattern or approach to problem-solving. Generally word and number clues serve as the predominant method of attack on problem-solving.[34] However, contradicting this claim, Studebaker, co-author of Study Arithmetic states:

> Great care has been taken throughout the book to avoid the use of clue words in solving problems. The substitution of a search for mechanistic clue words in problem solving for reasoning and understanding cannot be too severely condemned.[35]

Nevertheless, Kinney[36] suggests that traditionally the primary emphasis in the lower grades is commonly focussed on the answer since the problem-solving process is part of the program for learning the operations. As a result, Kinney[37] advances the thought that the pupil may

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[34]Clyde G. Cole,"Thought Processes in Grade Six Problems," The Arithmetic Teacher, 5: p. 202, October, 1958.

[35]J.W. Studebaker and others, Study Arithmetic, Book 3, Teacher's Guidebook with Answers,(Toronto: W.J. Gage and Company, 1949).

[36]Lucien B. Kinney,"Developing Ability to Solve Problems," The Mathematics Teacher, 52: 290-94, April, 1959.

[37]loc. cit.

not even be aware later, that there _is_ a general process for problem-solving. Singleton remarks that:

> Sometimes students become 'answer conscious' when solving word problems. They feel that the major goal is to find some magic number they can call the answer. Many times the choice of the fundamental operation is a guessing game, depending to some extent on the size and the position of numbers in the problem.[38]

From 1949 to 1962, the authorized elementary arithmetic series was _Study Arithmetic_.[39] Certainly the longevity of this authorization would classify the text if for no other reason as traditional. As a textbook classified traditional several interesting and significant points come to focus. The authors on the one hand profess:

> Experience has shown that children who are given definite instruction in deciding on the process to be used in solving a problem and in judging whether they have selected the right one make rapid strides in learning to solve problems. The child must have a pattern of thinking, a plan of attack, in order to solve problems successfully.[40]

Yet the authors of _Study Arithmetic_ from time to time and page to page ask these questions of the student.

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[38] Marilyn C. Singleton,"An Approach to Solving Problems," _The Mathematics Teacher_, 51: 212, March, 1958.

[39] J.W. Studebaker and others, _Study Arithmetic_, Books 3 to 6,(Toronto: W.J. Gage and Company, 1949).

[40] J.W. Studebaker and others, _Study Arithmetic Teacher's Guidebook_, _Grade 3_,(Toronto: W.J. Gage and Company, 1949), p. 11.

"What must I find? Shall I subtract? Add? Multiply? Divide? Why? What numbers shall I use?"[41] In the grade three text one finds:

> Is there a number that you do not need in finding the answer? What is it? Watch out for numbers that you don't need when you are finding answers to problems. Use these four questions to help you find answers to problems. (1) What is the question to the problem? (2) Should I add or subtract? (3) What numbers do I use to find the answer? (4) What is the answer for the problem?[42]

These and many other morsels of advice are conveniently placed in the textbook.

Generally, then, no concrete problem-solving approach is indicated in the traditional textbook of arithmetic. Kinney[43] again found that most textbooks provide a series of steps in problem-solving which the pupil may be taught to follow and on which he can be tested from time to time. Research findings, he proposed, provided some support for the practice, indicating that while any systematic procedure is better than none at all, no one series of steps would be better than any other series.

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[41] Study Arithmetic Book 6, p. 140.

[42] Study Arithmetic Book 3, p. 202.

[43] Kinney, op. cit., p. 290.

Buswell,[44] in his study previously cited, concluded that the evidence gave no support to the notion that problem-solving must follow the neat precise recipes that are so frequently encountered in textbooks on methodology. Buswell concludes with a bold statement:

> One is forced to the conclusion that either (a) great variety in the process of problem solving is normal and to be accepted or (b) schools have been ineffective in teaching a technique of problem-solving, or, as often expressed "teaching the students how to think."[45]

In the limited time of seventeen years in education the investigator has come to know to some extent the apparent general pattern of sequential steps in problem-solving at the time the traditional textbook was utilized. A few examples of problem-solving patterns will indicate the more traditional approach. Generally teachers were concerned about order and rationale in trying to relate mathematics and problem-solving to a scientific method. Since teachers found no uniform or actual direction from the textbook, the teachers often developed the statement approach for students whereby students could satisfy the teachers that their think-on paper at least would be systematically planned. A typical problem of the one-step variety might be patterned thus:

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[44] Buswell, op. cit., p. 133.

[45] Ibid., p. 137.

Problem: A farmer raised an average of 17.84 tons of tomatoes an acre. How many tons did he raise on 8.5 acres?[46]

A statement for each number used in the binary operation is made with a concluding statement which includes the answer.

Statement I: The farmer raised an average of 17.84 tons per acre.
Statement II: The farmer raised 8.5 acres of tomatoes.
Statement III: The farmer raised 17.84x8.5 tons of tomatoes which is 150.64 tons.

The calculations related to the problem would normally be placed in a work area to the right of the statements.

A multiple-step problem might be expressed as follows:

Problem: A team won 18 games and lost 13 games. Express its "percentage" of wins as a three place decimal.[47]

Statement I: The team won 18 games.
Statement II: The team lost 13 games.
Statement III: The team played 31 games.
Statement IV: The percentage of wins is

$$\frac{18}{31} = .581$$

It should be noted that an extra statement is required because of the necessity of two operations or two steps in the problem. To what extent student's thought processes would be employed, the statement approach really does not answer except for the correctness of his process.

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[46]*Winston Mathematics Book 2.*, p. 77.

[47]*Winston Mathematics Book 1.*, p. 77.

THE MODERN PROBLEM SOLVING APPROACH

A review of the new textbooks on the market would seem to suggest a more definite approach to problem-solving. In particular, one series of texts now authorized, bases its problem-solving approach on the Gestalt psychology.[48] Rather than manipulating the parts to find the whole, the authors profess that an understanding of the whole precedes finding the parts.

Buswell[49] strongly felt that the results of his study had one decided implication for the schools. He suggested that in dealing with discovery generalizations, a definite lack of maturity in this kind of thinking was lacking. He felt that economy of time may necessitate the teaching of many principles expressed in abstract form, but some amount of attention to finding generalizations is necessary if students are to cope with problems for which discovery of a generalization is a cue to the solution.

Bruner would agree that "the teaching and learning of structure, rather than simply the mastery of facts and techniques is at the centre of the classic problem of

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[48]Henry Van Engen and others, Seeing Through Arithmetic, Books 3 to 6, (Toronto: W.J. Gage Limited, 1959).

[49]Buswell, op. cit., p. 137.

transfer."[50] According to Bruner, the main purpose in learning is that it will be of some utility in the future. This is accomplished in two methods of transfer; (1) specific transfer of skills, habits, associations or training and (2) non-specific transfer in the form of ideals, principles and attitudes.[51] He continues:

> In essence it consists of learning initially not as a skill but a general idea, which then can be used as a basis for recognizing subsequent problems as special cases of the idea originally mastered. This type of transfer is at the heart of the education process--the continual broadening and deepening of knowledge in terms of basic and general ideas.[52]

Lankford[53] also maintains that the effective teacher of mathematics encourages creativity by being a catalyst in the discovery of basic ideas, laws, or principles of mathematics.

Throughout the first half of the twentieth century psychologists have been somewhat active in the development of theories of learning. Most of this period felt the impact of Thorndike's stimulus-response explanation of learning. This psychology had extensive influence on educational

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[50]Jerome S. Bruner, The Process of Education, (Cambridge: Harvard University Press, 1962), p. 12.

[51]Ibid., p. 17.

[52]loc. cit.

[53]Francis G. Lankford Jr., Implications of The Psychology of Learning for The Teaching of Mathematics, (National Council of Teachers of Mathematics, 21st. Yearbook, 1955), p. 405.

practice even as it does today. Dominant within this theory were the laws of readiness, exercise and effect.[54] Now perhaps the most serious challenge to the stimulus-response view of learning came from the camp of Gestalt. For over three decades this theory has intrigued educators and increasingly it has replaced Thorndike's influence upon curriculum design and methodology. Particularly, this influence has been felt strongly in the field of mathematics.[55]

There are three basic propositions underlying this Gestalt psychology:[56] (1) all experience or mental activity implies a differentiation of the perceptual field to which an organism can react to some kind of figure-ground pattern; (2) the course of mental development is from a broad, vague and indefinite total to the particular and the precise detail; and (3) the properties of parts are functions of the whole system in which they are embedded.

In the view of learning held by the Gestaltists, insight is an important element. The process of making an organism aware of the conditions governing the phenomena to which it is reacting is essentially what is meant by the

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[54] _Ibid._, pp. 405-06.

[55] _Ibid._

[56] George W. Hartman,"Gestalt Psychology and Mathematical Insight," _The Mathematics Teacher_, 59: 656-61, November, 1966, pp. 657-58.

insight experience.[57] It seems that less dependence is placed upon repetitive practice in the Gestalt view of learning than in the Stimulus-Response (often called connectionist) theory. Further, much is made of the relationships that exist in the elements surrounding a learning situation which is the field. Thoughtfully making interpretations or analyses leading to insights into principles or laws or into solutions of problems is an essential of learning for the Gestaltists.[58]

The fundamental precept upon which the Gestalt approach is made is the structure of a concept. Lindstedt maintains that to get to the heart of any problem one must get at the structure of the problem situation.[59] No two learning situations can be alike in all respects, for the likeness is found in the organization of the whole and not in the substance of the pieces.[60] Upon this fundamental concept, structure, is the rationale of the new Gestalt approach to problem-solving.

Hartung and Van Engen, co-authors of the present

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[57] *Ibid.*, p. 660.

[58] Lankford, *op. cit.*, p. 406.

[59] S.A. Lindstedt, "The Problem of Solving Arithmetic Problems," (W.J. Gage Limited, lithographed), p. 23.

[60] Hartman, *op. cit.*, p. 661.

authorization, Seeing Through Arithmetic, claim that at the grade two level there are two readiness steps to be taken in preparing the elementary school child for problem-solving:

> (1) it is possible to prepare pupils for the idea that a verbal problem is a description of something that has been or could be done; (2) it is necessary to establish with pupils the habit of visualizing a problem situation.[61]

They further advance that every problem has structure. Lindstedt reiterates that every problem has some kind of action, that there is something going on. The action may be physical, it may be imagined but most significantly these actions, in turn can be related to specific mathematical models or structures.[62] This structure for problem-solving can be visualized as a square according to Lindstedt. The left-hand side is found in the world of physical events, where ideas abound and problems arise. The right-hand side is in the abstract mathematical world of symbolization and computational skill. The conceptual model as designed by Lindstedt is detailed on the following page.[63]

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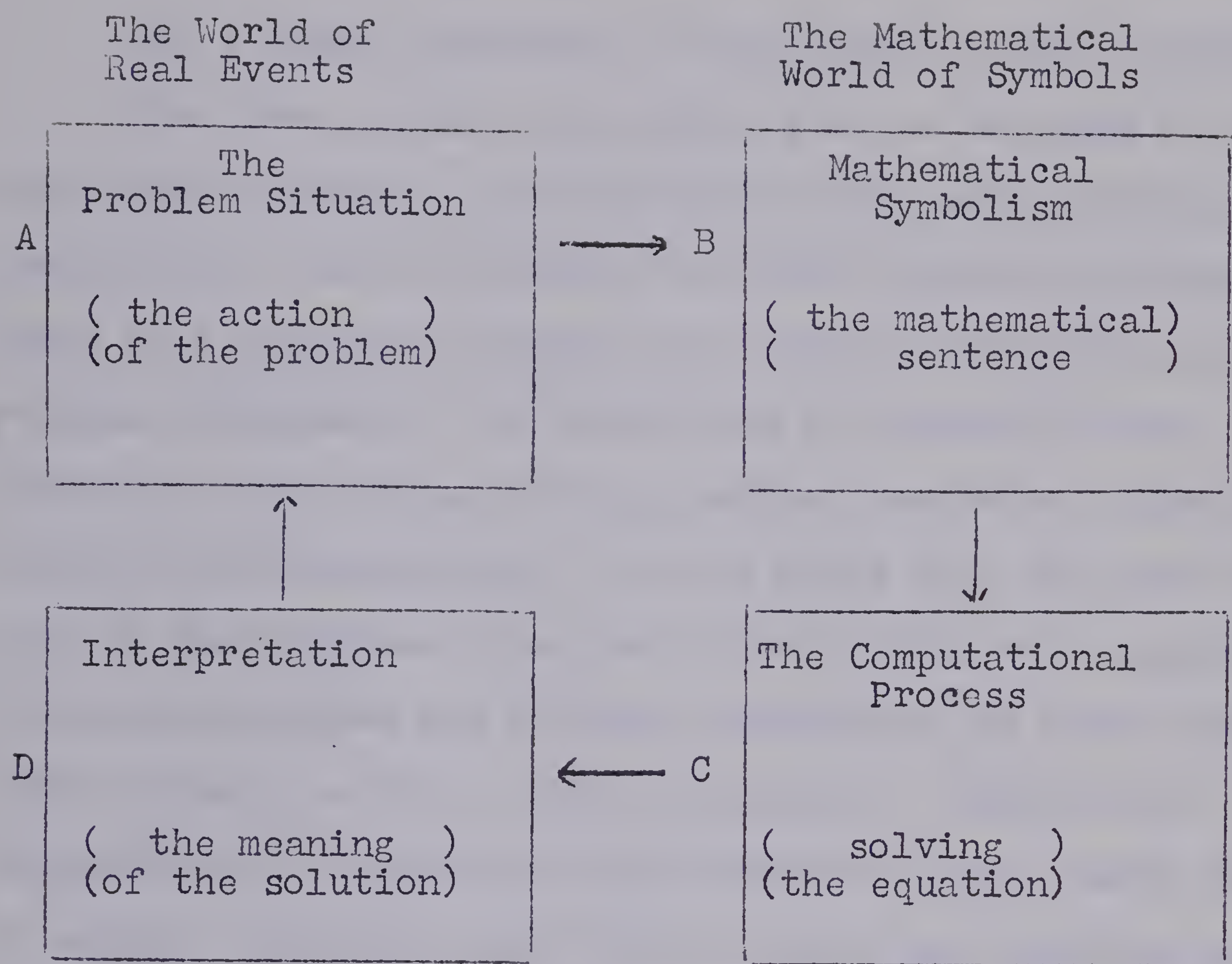
[61]Maurice L. Hartung and others, Charting The Course for Arithmetic, (Chicago: Scott, Foresman and Company, 1960), p. 33.

[62]Lindstedt, op. cit., p. 16.

[63]Ibid., pp. 25-26.



The following five steps are repeatedly employed in the *Seeing Through Arithmetic* series to provide a basis for problem-solving attack:

(1) Visualization of the structure of the problem, which is attempting to understand the situation or action taking place or which could take place.

(2) Translating the analysis of the situation into a mathematical statement. The statement must simply relate the story as it exists in the problem. (an unknown or placeholder is used at the grade two level)

(3) Solving the equation.

(4) Checking of both the equation with the answer as well as the logic of the answer in relation to the problem situation.

(5) A verbal statement of the answer to the question.

The above sequence of steps is to be employed in solving every problem. In the series of six books through grades one to six the authors introduce eighteen problem types in a sequence according to level of structure and problem situations. The situations are classified as: additive, subtractive, multiplicative, quotitive, partitive, divisive, and comparative. At the grade five and six level, nine of the eighteen types are solved by the rate equation. All multiplicative and divisive situations and many comparative situations have a ratio structure. Students are systematically exposed to this structure in the early years of school. Students must learn to solve and practice solving eighteen types of equations which result from the eighteen possible situations.[64]

> Example: John had some apples. Tom gave him 3 apples. Then he had 12 apples. How many apples did he have to begin with?

The situation is decidedly one of combining, therefore it is referred to as additive. John had some apples. He received more.

(1) Situation: Additive, combining of groups

(2) Equation: Using the screen, $\boxed{}$, or 'n' as the placeholder the student would write:

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[64] Hartung, op. cit., pp. 170-72

n	$+$	3	$=$	12
(The apples John had)		(Apples Tom gave him)		(Total apples he had)

The equation or the mathematical model is the same story as the problem. This is the whole key to the thinking in problem-solving as proposed by these authors.

(3) Computation: What number added to 3 produces 12? The computation is different from the situation.

$$n = 12 - 3 = 9$$

(4) Check: 9 apples plus 3 apples is 12 apples.

(5) Statement: John had 9 apples at first.

The foregoing problem is introduced in the third grade. At the other end of the elementary grades the more advanced problem type at the grade five level is presented.

Example: John has 12 oranges. He has 3 times as many apples as oranges. How many apples has he?

(1) Situation: Comparative and hence rate. The student must gain insight into the rate of 3 apples for every orange.

(2) Equation: A ratio equation is formed.

$$\frac{3}{1} = \frac{n}{12}$$

(rate of 3 apples for every orange)	(rate of n apples for 12 oranges)

(3) Computation: Using the ratio test or the equivalence of fractions the student obtains:

$$1 \times n = 3 \times 12$$

$$n = 36$$

(4) Check:

$$\frac{3}{1} = \frac{36}{12} \; ; \; \frac{3}{1} = \frac{3}{1}$$

(5) Statement: John has 36 apples.

A Comparison of The Two Problem-Solving Approaches

Applying these two approaches to the problem example of page 27, we have:

> Problem: A farmer raised an average of 17.84 tons of tomatoes an acre. How many tons did he raise on 8.5 acres?

Traditional Approach	Modern Approach
The farmer raised an average of 17.84 tons per acre.	$\frac{17.84}{1} = \frac{n}{8.5}$
He raised 8.5 acres of tomatoes.	$n = 17.84 \times 8.5$
He raised 17.84 x 8.5 tons of tomatoes = 150.64 tons.	$n = 150.64$
	The farmer raised 150.64 tons of tomatoes.

These two approaches to problem-solving are the major consideration of this investigation.

Henderson states three steps in problem-solving:

> (1) orienting to the problem -- a process by which the organism grasps the material of thought and keeps it available for deliberation;
>
> (2) producing relevant thought material - perception, concepts, generalizations;
>
> (3) forming and testing hypotheses.[65]

The sequence of steps proposed by Hartung and Van

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[65]Henderson and Pingry, op. cit., pp. 229-30.

Engen are really nothing more than that stated on the previous page by Henderson and Pingry or of Bingham[66] cited on page 24.

The fundamental process, then, of the new approach to problem-solving is to have the child realize the structure of the situation presented by the problem. Having formulated insight into the makeup of the problem, the student can then, and only then, translate this insight into a mathematical model or statement which results in an exercise of computation for the student.

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[66]Bingham, op. cit., p. 13.

CHAPTER III

THE EXPERIMENTAL DESIGN AND STATISTICAL PROCEDURES

The purpose of this study is to investigate the following questions in a careful but unsophisticated manner. Do students studying the Gestalt-ratio approach to problem-solving in grades 6, 7, and 8 achieve better in general problem-solving proficiency than students studying a traditional approach? Do these same students achieve better on rate problem types than the traditional students? Do Gestalt-ratio problem-solvers achieve better in solving non-rate problem types than the traditional students? Which method of solving problems, traditional or modern, requires greater time to solve problems in an orderly and systematic solution? Do the Gestalt-ratio students do better on a general mathematics test at the end of grade nine than the traditionally taught students?

Further, can the study show a reasonable description which would compare the apparent effects of the treatments on proficiency in multiple-step and single-step problem-solving? Also the study will look at the relationships of time and achievment as well as sex and achievement.

Background Leading to The Experiment

In September, 1961 the County of Beaver received authorization to experiment with the new mathematics. Two series of textbooks were used, Arithmetic We Need[1] and Seeing Through Arithmetic.[2] The problem-solving approach presented in the Arithmetic We Need series was upon further investigation much more traditional compared to the new Gestalt and ratio approach presented in the other series. To maintain proper perspective in problem-solving the Arithmetic We Need students were taught the situation-equation approach in accordance with Seeing Through Arithmetic and whenever discrepancies in approach occurred, the Seeing Through Arithmetic text became the authority. It was soon noticed that the greatest discrepancy was in problem-solving. Consequently all students regardless of series studied, were taught the new approach.

Teacher acceptance of the Seeing Through Arithmetic series as being superior warranted the implementation of this series in all elementary classes of the County in September, 1962. However, no change in authorization was effected in the junior high school for the ensuing year.

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[1]Guy T. Buswell and others, Arithmetic We Need, Bk. 3-6, (Toronto: Ginn and Company, 1960).

[2]Henry Van Engen and others, Seeing Through Arithmetic, Books 3 to 6, (Toronto: W.J. Gage Limited, 1959).

Therefore students in grade six classes studying the new mathematics for the first time in the school year 1961-62 would progress to grade seven in 1962-63 with no new authorization. Consequently the investigator prepared a teacher's guide for a modified program in Winston Mathematics[3] which would attempt to maintain the concepts established in the grade six year, especially the approach to problem-solving. A similarly prepared guide for the grade eight year based upon the same philosophy assisted the teachers in presenting the new mathematics using again the traditional text, Winston Mathematics, Intermediate 2, for the conceptual framework.

In a September seminar of each experimental year the investigator instructed the participating teachers regarding the new philosophy and approach. Frequent visits to the experimental classrooms maintained constant supervision of the program. Excerpts from the teaching guides are included in Appendices B and C. Each group in grade nine studied Mathematics for Canadians, Book 1.[4]

Subjects Included in The Study

The control group (TT) consisted of 128 students who

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[3]H.L. Stein and others, Winston Mathematics, Intermediate 1 and 2, (Toronto: Holt Rinehart and Winston of Canada, Limited, 1954).

[4]Henry Bowers and others, Mathematics for Canadians, Book 1, (Toronto: M.M. Dent and Sons, Canada, Limited and The Macmillan Company of Canada Limited, 1947).

completed grade eight in June, 1963. The experimental group (MT) consisted of 139 students who completed grade eight in June, 1964. Table I shows the distribution of students according to the two teaching methods. In each case the same schools participated in the study. The same teachers taught the TT and MT treatment groups in grade 7 and in grade 8. Table II indicates the progress by year and grade of the control group TT over a period of 9 school years. Table III presents similarly by year and grade the progress of the experimental group MT.

TABLE I

THE NUMBER OF CLASSES, STUDENTS, SCHOOLS, AND TEACHERS IN EACH PROGRAM APPROACH

Program	Number of Classes	Number of Students	Number of Schools	Number of Teachers
TT	6	128	3	3
MT	6	139	3	3
	12	267	6	6

TABLE II

MATHEMATICS INSTRUCTION OF CONTROL GROUP (TT)
BY YEAR, GRADE AND TEXTBOOK STUDIED

Year	Grade	Textbook Studied	
1955-56	1	Making Sure of Arithmetic	1
1956-57	2	Making Sure of Arithmetic	2
1957-58	3	Study Arithmetic	3
1958-59	4	Study Arithmetic	4
1959-60	5	Study Arithmetic	5
1960-61	6	Study Arithmetic	6
1961-62	7	Winston Mathematics	1
1962-63	8	Winston Mathematics	2
1963-64	9	Mathematics for Canadians	1

TABLE III

MATHEMATICS INSTRUCTION OF EXPERIMENTAL GROUP (MT)
BY YEAR, GRADE AND TEXTBOOK STUDIED

Year	Grade	Textbook Studied	
1956-57	1	Making Sure of Arithmetic	1
1957-58	2	Making Sure of Arithmetic	2
1958-59	3	Study Arithmetic	3
1959-60	4	Study Arithmetic	4
1960-61	5	Study Arithmetic	5
1961-62	6	Arithmetic We Need or Seeing Through Arithmetic	6
1962-63	7	Modified Winston Mathematics	1
1963-64	8	Modified Winston Mathematics	2
1964-65	9	Mathematics for Canadians	1

Six schools each year used the materials outlined in Tables II and III. For the purpose of the investigation, three schools were selected on the following basis. Geographically the schools represented an agricultural community a short distance southeast of Edmonton. Six schools taught at least grades one to nine. For administrative feasibility and to procure a reasonable cross-section of the County population, three schools were selected, two schools, one at each extremity, and one in the centre. In these three schools classroom characteristics for statistical purposes were the most satisfactory combinations.

Special Modified Program in Grade Seven and Eight

Excerpts of the more pertinent parts of A Guide for A Modified Program in Grade Seven Mathematics for Use with Winston Mathematics Book 1, 1962-63 and the sequel edition, A Guide for A Modified Program in Grade Eight Mathematics for Use with Present Winston Mathematics Book II, 1963-64, appears for convenience in Appendices B and C. The general purposes of the modified program were: (1) since the grade seven mathematics course as authorized by Winston Mathematics, Bk. I contained topics or parts of topics which were either concepts previously covered in grade six or approaches not compatible with the new mathematics in grade six, certain definite treatments needed to be outlined for the benefit of the grade seven teacher; (2) the new approach,

Gestalt, to problem-solving as presented in the Gage program in grade six required maintenance; (3) the concepts of ratio and the properties of the number system and other general concepts needed to be continued and maintained in the junior high school; and (4) to fill a void created by an absence of a curriculum guide in junior high school mathematics patterned in the new mathematics.

The guides were prepared by the investigator for each teacher of the program. Anticipated problem areas of instruction were provided. The guide systematically covered the *Winston Mathematics* rather thoroughly page by page outlining the kind of treatment the teacher should employ on any given topic by page. Numerous hints on how to present concepts and on how to solve problems using and maintaining the Gestalt-ratio approach were included.

Problem Solving Test Eight (PS8)

The *Problem Solving Eight Test*, PS8, was designed for the study. The investigator studied examples of several commercial standardized mathematics tests but few if any met the requirements of the testing experiment -- a test which would provide space for the required statement or equation, the calculations and the answer. Items similar in language difficulty, mathematical understanding and mathematical type to the *Winston Mathematics* were designed.

The items were as well as possible graded (1) in language and situation; (2) by arithmetic process involved in the problems, and (3) in relationships and reasoning in a problem situation. To provide a social background, the test is divided into three social areas: vacation on the farm, chickens of the farm, and the city and the farm. Following each of these sections are several problems which relate to the social situation. Being a rural school system, the farm social setting was believed reasonable justification for the format of the test. The complete test is provided in Appendix A.

No pre-run was attempted because the types of questions comprising the test were analogous to problem types found in Winston Mathematics. Since both treatment groups studied the same problem types from the same text but under different treatments, the investigator felt justified in employing the special test.

In order to analyze the effect of time as a variable in the study, no time limit was imposed. Students were required to either write statements or equations for their solutions to each problem in the space provided. Space for calculation and the answer was provided. In this manner the student was encouraged to present an orderly and systematic solution to his problem. Upon completion of the test the student entered in the space provided the time in minutes

required to write the test.

The Experimental Procedure

As indicated in Table II and Table III on pages 44 and 45, each group proceeded through the grades according to the curriculum outlined. In June of each group's final year in grade eight the Otis Quick-Scoring Test of Mental Ability, Form Beta EM,[5] and the Problem Solving Test Grade Eight were administered. Thus the (TT) group wrote each test in June, 1963 and similarly the (MT) group in June, 1964. The following year in grade nine each group wrote the Grade IX Mathematics Departmental Examination, in 1964 and 1965 respectively. The investigator in his experimental design wished to follow the effect, if any, of the modified grade seven and eight program into grade nine which was once again a traditional program.

Though the mathematics test in grade nine is designed to test mathematical understandings and concepts, computational skills, and problem-solving, as compared to the PS8 test which is basically a problem-solving test first and a computational test second, the investigator felt justified in employing this test on the following basis.

The modified program in grade seven and eight maintained mathematical concepts and understandings developed

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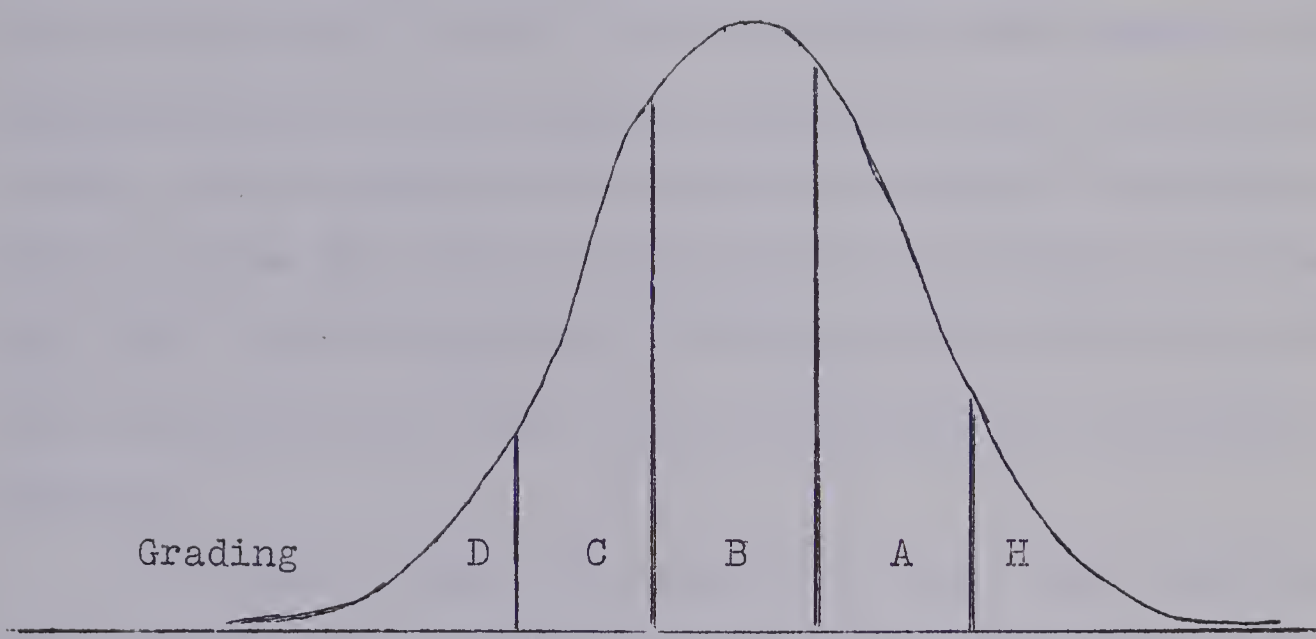
[5]Arthur S. Otis, Otis Quick-Scoring Mental Ability Tests: New Edition, Beta EM, (New York: Harcourt, Brace & World, Inc., 1954).

in grade six through Seeing Through Arithmetic, and presented those concepts common to Winston Mathematics in grades seven and eight. The concepts taught were in the final analysis the same, the difference only in approach. Since both groups studied the same text in grade nine and prepared for the usual stereotyped examination, teachers of grade nine likely did not teach differently to the two groups. What, then was the effect of the (MT) teaching?

Certain obvious limitations must be accepted in using the data from the grade nine mathematics examination. It is clear that each grade nine group did not write the same examination. Hence raw score data would be invalid. However, since data each year is subjected to the normal curve, only data based on relation to normality was considered feasible for the study. The investigator chose stanine scores based on the distribution of the normal curve for the province as a whole. This distribution is shown in Figure I. A definite amount of information is obviously lost by using stanine scores, but under the circumstances the invalidity of the raw score presented no alternative. Further, the M9 test was a timed test as compared to the untimed PS8 test. With these limitations recognized the grade nine data was employed in the study.



Stanines	1	2	3	4	5	6	7	8	9
Percentage	4	7	12	17	20	17	12	7	4

FIGURE 1

GRAPHIC ILLUSTRATION OF THE RELATIONSHIP
BETWEEN STANINE SCORES AND NORMAL CURVE

Statistical Procedures

In order to be able to study the resultant effects on achievement by ability groups, a frequency distribution of OTIS scores was made for each treatment group. Ability levels were arbitrarily made on the basis of possible school success. Thus students obtaining an OTIS score of 116 or more were considered high (HI); students obtaining a quotient between 106 and 115 (all inclusive) were denoted high average (HA); students scoring between 96 and 105 (inclusively) were considered average (AV); students obtaining a score between 86 and 95 (inclusively) were denoted low average (LA); and the remaining students scoring less than 86 were considered low (LO). This distribution is presented in Table IV.

To better study the effects of time, time intervals were arbitrarily established from a frequency distribution of time scores. Consequently, the following time level groups were established. Fast workers (FW) were the group of students who required 79 minutes or less to complete the PS8 test; average workers (AW) would be the group requiring between 80 and 102 minutes (inclusively); and the slow workers (SW) the group of students needing 103 minutes or more to complete the PS8 test. Table V shows this distribution in each category.

TABLE IV

NUMBER OF STUDENTS IN EACH OTIS ABILITY CELL IN EACH TREATMENT GROUP

	Ability					
Group	HI	HA	AV	LA	LO	GROUP
TT	18	38	41	22	9	128
MT	21	49	35	24	10	139
	39	87	76	46	19	267

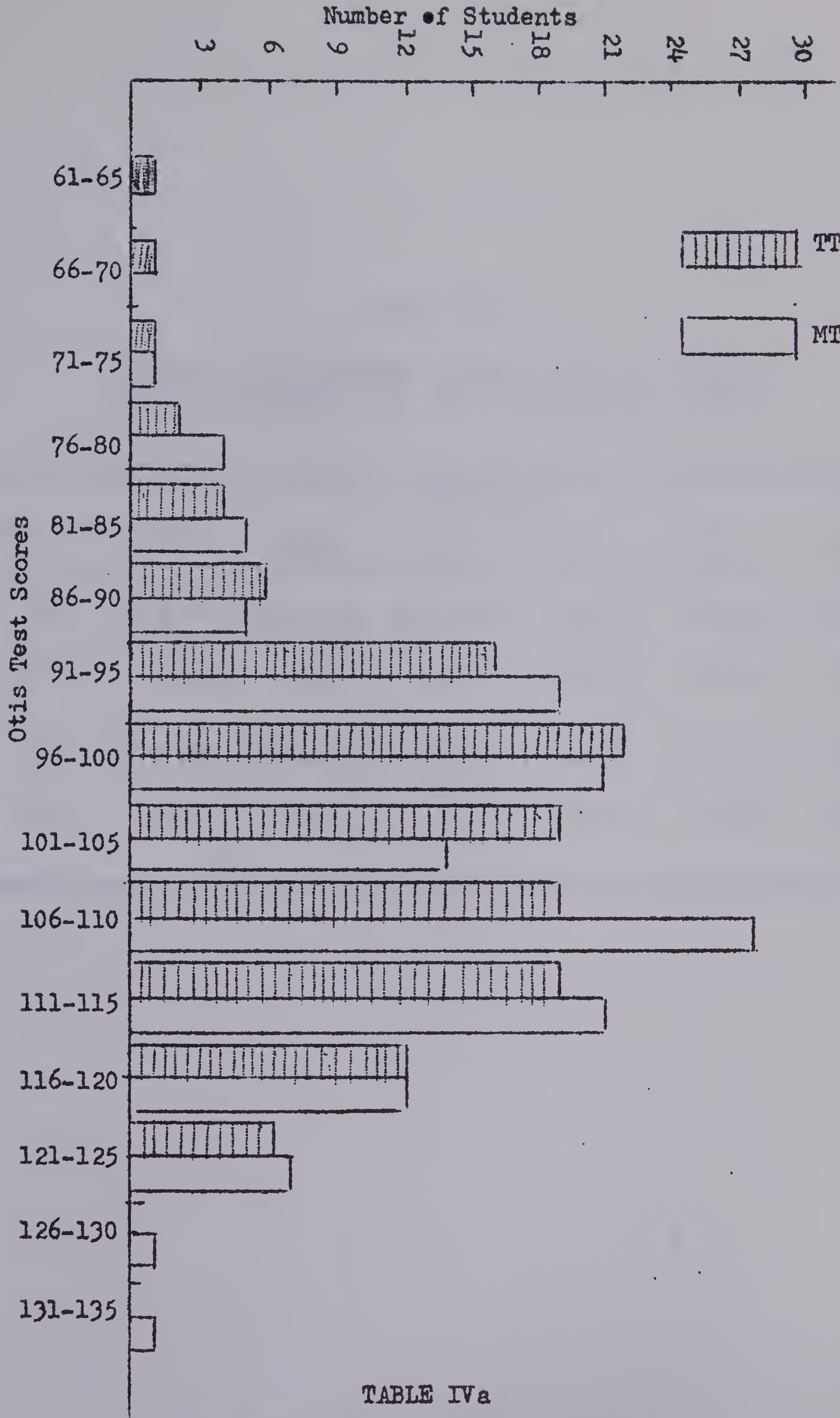


TABLE IVa

FREQUENCY DISTRIBUTION OF OTIS TEST SCORES FOR EACH TREATMENT GROUP

TABLE IVb

MEANS AND STANDARD DEVIATIONS OF OTIS ABILITY GROUPS FOR EACH TREATMENT GROUP

Group		Ability					
		HI	HA	AV	LA	LO	GROUP
TT	MN	119.83	110.32	100.29	92.05	77.11	102.73
	SD	2.79	2.90	2.71	2.46	7.00	14.39
MT	MN	120.57	110.55	99.74	91.96	80.60	103.91
	SD	4.04	2.91	2.53	2.17	10.64	12.90

TABLE V

NUMBER OF STUDENTS IN EACH TIME GROUP
CELL IN EACH TREATMENT GROUP

Group	Time FW	Time AW	Time SW	GROUP
TT	25	82	21	128
MT	78	44	17	139
	103	126	38	267

TABLE VI

NUMBER OF STUDENTS IN EACH PROBLEM SOLVING ACHIEVEMENT GROUP IN EACH TREATMENT GROUP

Group	Achievement			
	PPS	APS	GPS	GROUP
TT	16	74	38	128
MT	25	71	43	139
	41	145	81	267

Problem-solving achievement groups were also established. Poor problem-solvers (PPS) were considered the group who scored between 0 and 8, inclusive, on the problem test; average problem-solvers (APS) were designated the group who scored between 9 and 17, inclusive, on the PS8 test; and good problem-solvers (GPS) were those who scored between 18 and 27, inclusive, on the PS8 test. The distribution of students in each category is shown in Table VI.

Description of Analysis Format

The two-way unweighted means analysis of variance as described by Ferguson[6] and Wert[7] was employed in the study. Being a very robust test of significance, the analysis of variance test need not require strict adherence to the assumption of normality and homogeneity of variance. The subjects of the study were selected from the same schools in the same geographical area in each year. The large numbers in the sample of 267 students would likely be representative of the population.

The primary purpose of employing the two-way unweighted means analysis of variance was to present to the reader a

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[6]George A. Ferguson, Statistical Analysis in Psychology and Education, (New York: McGraw-Hill Book Company, Inc., 1959), pp. 242-62.

[7]James E. Wert and others, Statistical Methods in Education and Psychological Research, (New York: Appleton-Century-Crofts, Inc., 1954), pp. 188-207.

reasonably systematic and orderly presentation of data which might provide a useful format in designing similar studies in rural and small jurisdictions of comparable size to the County of Beaver. Thus, it is certainly recognized that the assumptions of randomness and independence are limitations to be considered, and hopefully ones which would be treated in other studies. Because of the 'time-lapse' nature of the samples in this study, the assumptions of randomness cannot be met and the assumption of independence of the two samples is assumed but not tested.

Where the two-way unweighted means analysis of variance was not applicable, only descriptive statistics were employed to convey a description of the results of the treatments. Tables of means and profiles were thus employed for this purpose.

Upon scrutiny of the significant F-ratios following the application of the two-way unweighted means analysis of variance which indicated further analysis, the t-test as described by Ferguson[8] was applied to each individual pair of ability group cell means to determine if significant differences existed between treatment groups. Estimated unbiased variance used for the t-test was determined from raw data using the computational formula described by Guilford.[9]

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[8]Ferguson, op. cit., pp. 136-38.

[9]Joy Paul Guilford, Fundamental Statistics in Psychology and Education, 4th. Edition, (New York: McGraw-Hill Book Co., 1954), p. 94.

Variance was pooled for the purpose of the analysis to better achieve the best unbiased estimate of the population variance of the pairs of groups being tested. Appropriate significance tables in Ferguson[10] were consulted for determining the critical t-value. The critical value for the level of significance used in the study was the probability of 1 out of 100 that the observed difference in means could result from sampling error.

All data was punched on IBM cards and the computer was used to obtain a more accurate analysis. However, if this study were to be replicated in any form a calculator of the type used in most school division offices would suffice for accuracy and facility.

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[10]Ferguson, op. cit., pp. 308-13.

CHAPTER IV

THE RESULTS OF THE INVESTIGATION

Findings from The Statistical Analysis

For simplicity of reporting in this chapter, each of the null hypotheses tested is stated immediately before the presentation of the results of the statistical tests employed. A brief and simple interpretation follows the presentation of each set of results.

Analysis of Variance of PS8 Test Scores

Null Hypothesis I

> On the PS8 test there are no significant differences (a) between the group mean scores obtained by the TT and MT students,
> (b) among the ability level mean scores,
> (c) attributable to interaction.

Table VII displays the PS8 test cell means and Table VIII summarizes the analysis of variance carried out on the PS8 test scores.

Since the observed F-ratio (0.0069) for comparison between group mean PS8 scores was decidedly less than the critical F-ratio (6.75), Null Hypothesis I a) was not rejected.

Since the observed F-ratio (42.59) for comparisons among the ability level mean PS8 scores exceeded the critical value (3.40), Null Hypothesis I b) was rejected.

TABLE VII

PS8 CELL AND GROUP MEANS

Group	Ability					
	HI	HA	AV	LA	LO	GROUP
TT	18.00	15.79	13.83	11.82	6.56	14.15
MT	19.67	16.45	11.09	11.79	7.00	14.10
				Grand Mean		14.13

TABLE VIII

SUMMARY OF ANALYSIS OF VARIANCE OF PS8 TEST SCORES

SOURCE	SS	DF	MS	F
Group	0.11	1	0.11	0.0069
Ability	2073.76	4	675.94	42.59
Interaction	179.18	4	49.80	2.82
Within	4079.11	257	15.87	--

$F_{.01}(1,257) = 6.75$ $F_{.01}(4,257) = 3.40$

Since the observed F-ratio (2.82) attributable to interaction did not exceed the critical value (3.40), Null Hypothesis I c) was not rejected.

No significant difference between the group(treatment) PS8 test mean scores was observed. However, a significant difference was observed, as expected, among the ability level PS8 test mean scores. The interaction effect was not statistically significant. Inspection of Table VII showed that in the MT treatment group the low average ability group scored higher than the average ability students. The difference, however, was not significant. Nevertheless, the observed difference was not expected to be in favour of the lower ability group.

Observation of Table VII and Figure 2 reveals an interesting description of the apparent effects of the two treatments in the study. Though the main effects were clearly not significant the F-ratio for interaction effect was higher (2.82) and such an effect can be observed in the profile in Figure 2. It would appear that the higher ability students, HA and HI, scored higher as a result of the modern treatment. The same pattern apparently existed for the lower ability student, LO. However, the profile indicates that possibly the average ability student, AV, may have encountered difficulty in mastering the modern approach to problem-solving.

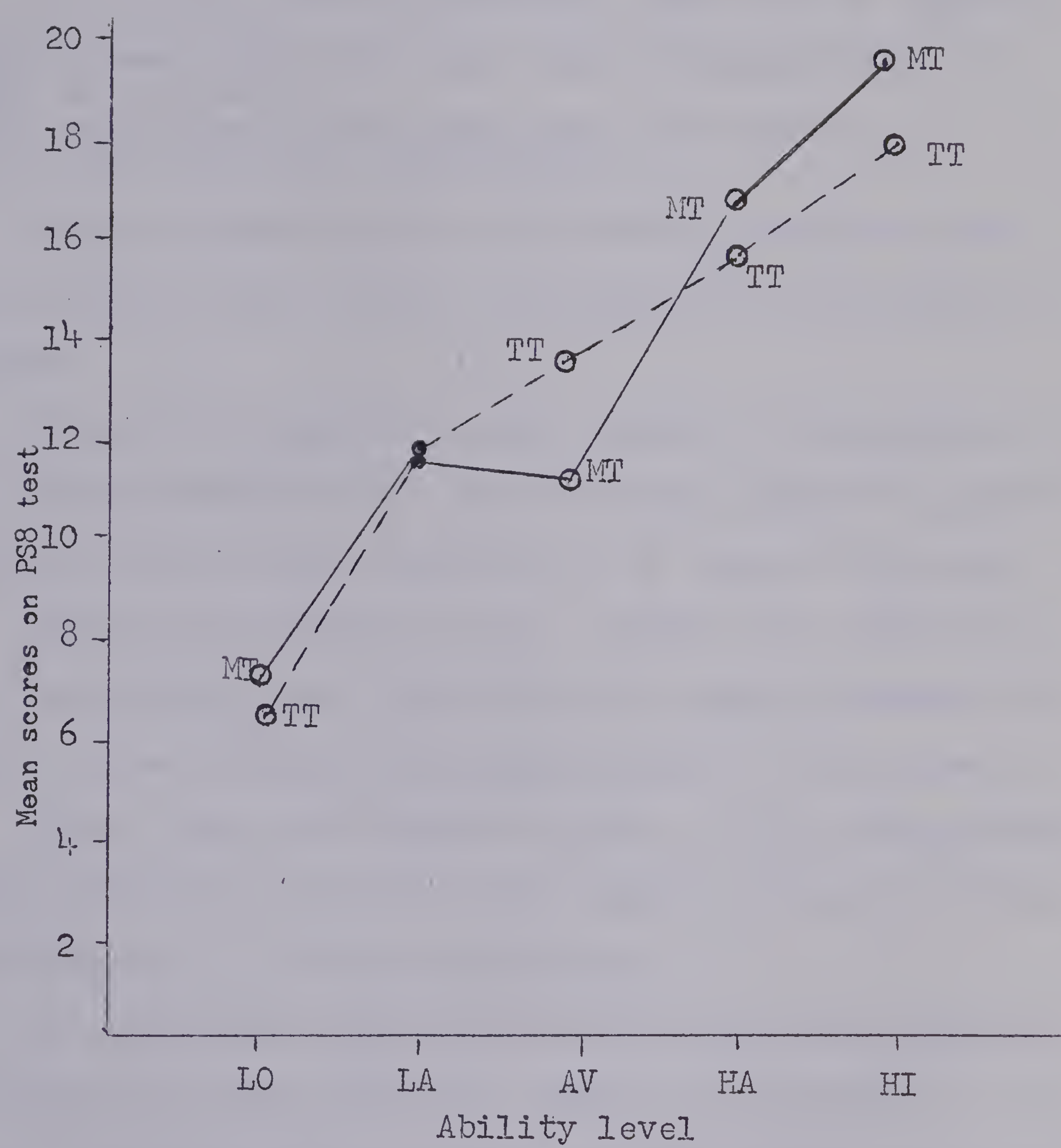


FIGURE 2

CELL MEANS PS8 SCORES FOR THE TWO GROUPS AT THE FIVE ABILITY LEVELS

Analysis of Variance PS8(Rate) Test Scores

Null Hypothesis II

> On the PS8 test(rate section) there are no significant differences
> (a) between the group mean scores obtained by the TT and MT students,
> (b) among the ability level cell mean scores,
> (c) attributable to interaction.

Table IX exhibits the PS8 test(rate section) cell means, whereas Table X shows the summary of the analysis of variance.

Since the observed F-ratio (0.001) for comparison between group mean PS8(rate) scores did not exceed the critical ratio (6.75), Null Hypothesis II a) was not rejected.

Since the observed F-ratio (49.664) for comparisons among the ability level mean PS8(rate) scores exceeded the critical value (3.40), Null Hypothesis II b) was rejected.

Also, since the observed F-ratio (1.713) attributable to interaction did not exceed the critical value of (3.40), Null Hypothesis II c) was not rejected.

No significant difference between the group PS8(rate) test mean scores was observed. However, as expected, a significant difference was observed among the ability level PS8 (rate) test mean scores. It was further observed that the interaction effect was not significant. Inspection of Table IX indicated again that in the MT treatment group the low average ability group scored higher than the average ability group. The difference was not significant.

TABLE IX

PS8 (RATE) CELL AND GROUP MEAN SCORES

Group	Ability					
	HI	HA	AV	LA	LO	GROUP
TT	13.50	12.03	10.26	8.68	4.44	10.56
MT	14.81	12.31	8.06	8.58	5.10	10.45
				Grand Mean		**10.51**

TABLE X

SUMMARY OF ANALYSIS OF VARIANCE OF PS8 RATE TEST SCORES

SOURCE	SS	DF	MS	F
Group	0.0097	1	0.0097	0.001
Ability	2059.83	4	514.96	49.664
Interaction	71.06	4	17.76	1.713
Within	2664.78	257	10.37	--

$F_{.01}(1,257) = 6.75$ $F_{.01}(4,257) = 3.40$

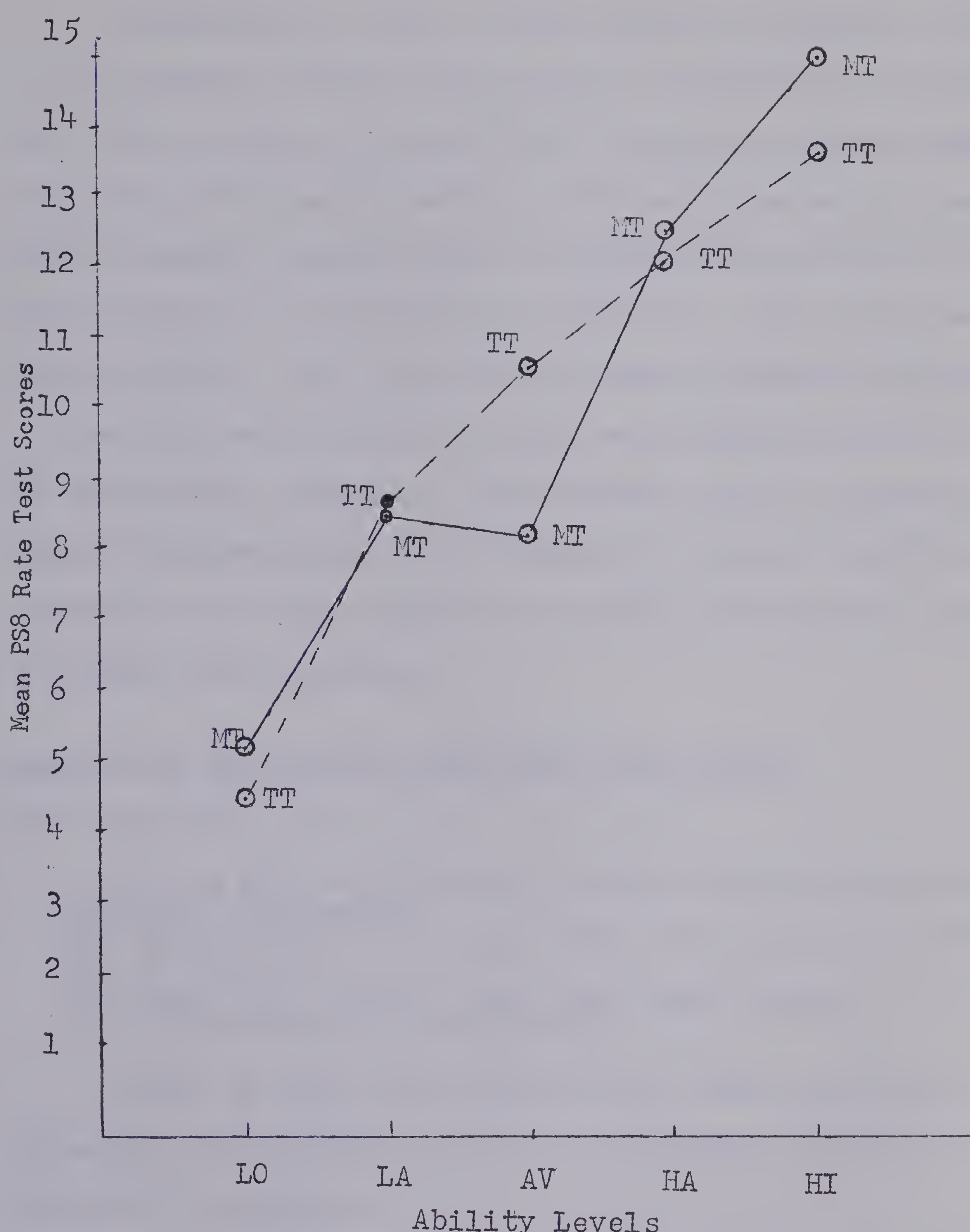


FIGURE 3

CELL MEAN PS8 RATE SCORES FOR THE TWO GROUPS AT THE FIVE ABILITY LEVELS

Inspection of Table IX and Figure 3 reveals a picture of the apparent effects from the two treatments. On the PS8 test, rate section, it seems that the above average students, HA and HI, studying the modern mathematics materials performed somewhat better than the corresponding group of students studying the traditional materials. Also the low ability student, LO, who used the modern materials appeared to have done better than the same corresponding group studying traditional materials. The pattern for the average ability student appeared to indicate a degree of difficulty encountered by these students attempting the modern approach to solving rate problems.

Analysis of PS8(Non-Rate Section) Test Scores

Null Hypothesis III

> On the PS8 test(non-rate section) there are no significant differences
> (a) between the group mean scores obtained by the TT and MT students,
> (b) among the ability level cell mean scores,
> (c) attributable to interaction.

Table XI shows the PS8(non-rate) cell and group mean scores and the companion Table XII presents a summary of the analysis of variance.

Since the observed F-ratio (0.004) for the comparison between group mean PS8 non-rate scores is less than the critical ratio (6.75), Null Hypothesis III a) was not rejected.

TABLE XI

PS8 NON RATE CELL AND GROUP MEAN SCORES

Group	Ability					
	HI	HA	AV	LA	LO	GROUP
TT	4.50	3.76	3.56	3.14	2.11	3.58
MT	4.86	4.14	3.03	3.21	1.90	3.64
				Grand Mean		3.61

TABLE XII

SUMMARY OF ANALYSIS OF VARIANCE OF PS8
NON RATE TEST SCORES

SOURCE	SS	DF	MS	F
Group	0.0079	1	0.0079	0.004
Ability	157.49	4	39.37	21.377
Interaction	6.03	4	1.51	0.818
Within	473.35	257	1.84	--

$F_{.01}(1,257) = 6.75$ $F_{.01}(4,257) = 3.40$

The observed F-ratio (21.377) for comparisons among the ability level mean PS8(non-rate) test scores exceeded the critical F-ratio (3.40) and thus Null Hypothesis III b) was rejected.

Variance attributable to interaction had an observed F-ratio (0.818) which was less than the critical value (3.40). Thus, Null Hypothesis III c) was not rejected.

Interpretation of this data would indicate that there was no significant difference between the treatment groups on the PS8(non-rate) test scores. However as expected, a significant difference was observed among the ability level PS8(non-rate) test mean scores. No significance may be attached to the effect of interaction. Inspection of Table XI showed once again the unusual fact that the low average ability group scored higher than the average group in the MT treatment group but the difference was not significant. Figure 4 graphically illustrates the test mean scores.

Table XI and Figure 4 provide information regarding the apparent results of the two treatments on each of the groups with respect to non-rate problem-solving performance. Once again the high average ability students, HA and HI, appeared to score somewhat better while studying the modern materials. However, contrary to previous patterns for the total test scores and the rate section scores, the low

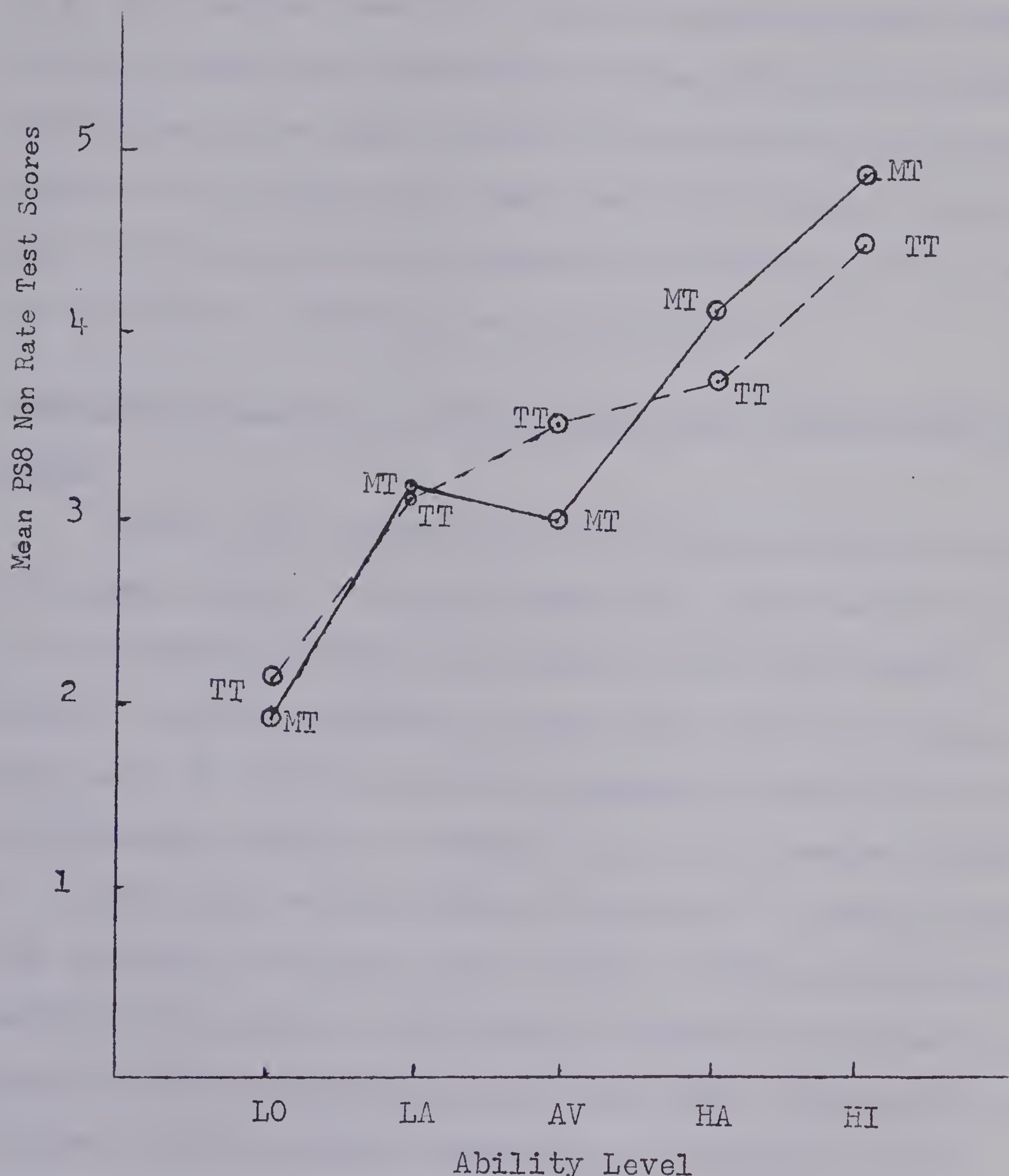
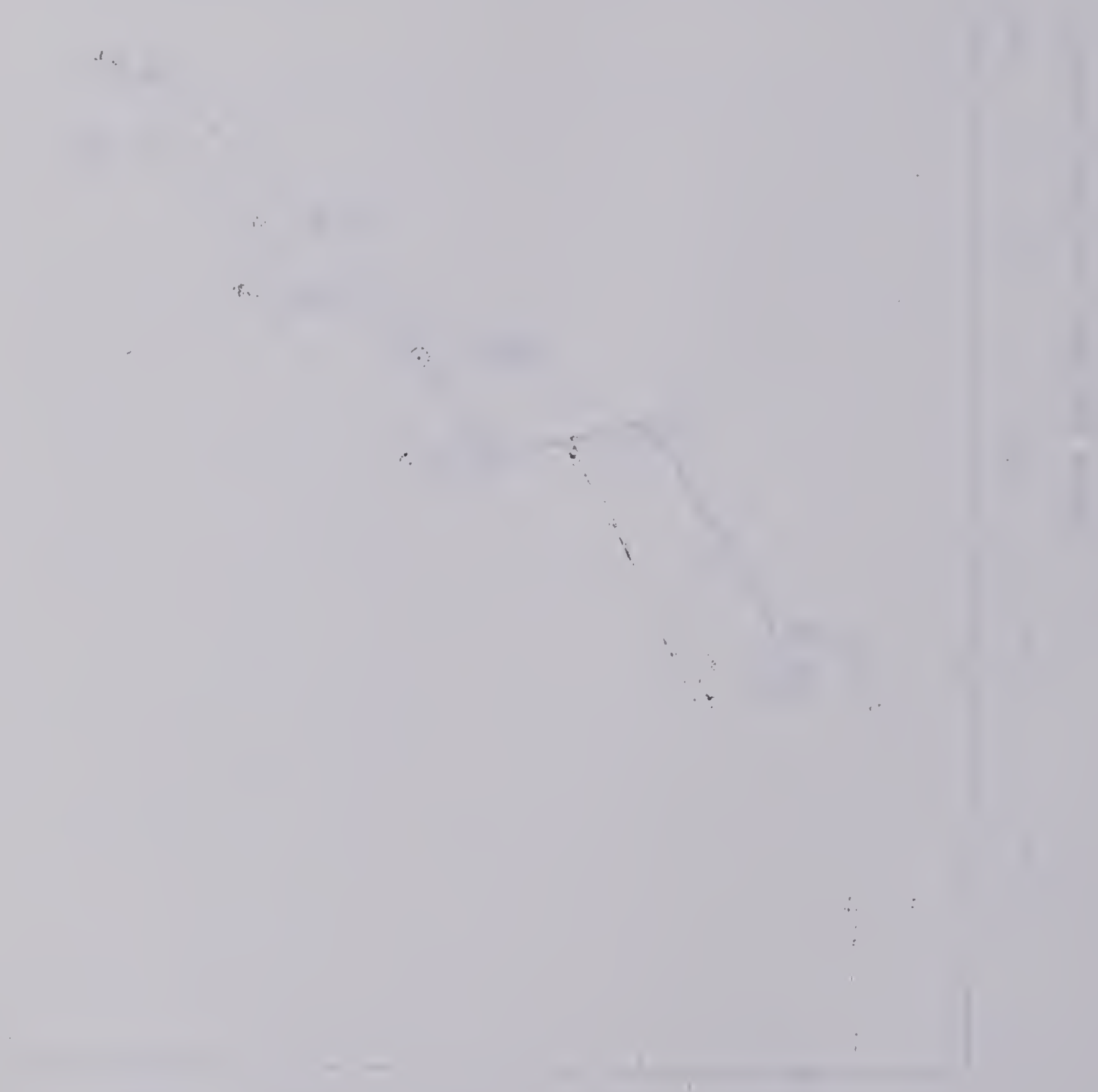


FIGURE 4

CELL MEAN PS8 NON RATE SCORES FOR THE TWO GROUPS AT THE FIVE ABILITY LEVELS



ability group students studying the modern materials appeared to have scored lower than the low ability group students studying traditional materials on non-rate problem types. Nevertheless, the same pattern for average ability students appeared to indicate that once again this ability level group did not apparently respond as effectively to the modern approach to teaching problem-solving.

Descriptive Analysis of The Multiple-Step Problem PS8 Test Scores

Table XIII displays the PS8 multiple-step cell and group mean scores. Figure 5 shows the cell mean PS8 multiple-step scores for the two groups at the five ability levels. Careful attention to the above table and figure would seem to indicate that the apparent success of solving multiple-step problems favoured the above average, HA and HI students who studied modern materials. A rather irregular pattern is evident. The average ability student who received the modern Gestalt-ratio treatment once again appeared to have responded poorer than their counterparts who followed the traditional materials. A seemingly small difference in favour of the low average ability students who studied the modern materials seemed possible. However, the low ability students, LO, apparently found difficulty with the modern approach to problem-solving particularly with

multiple-step problems. Observed as well was the apparent fact that in the Gestalt-ratio treatment group the average ability group scored poorer than the next lower ability level group, the low average students.

TABLE XIII

PS8 MULTIPLE STEP CELL AND GROUP MEAN SCORES

Group	Ability					
	HI	HA	AV	LA	LO	GROUP
TT	13.17	10.89	9.27	7.86	4.11	9.70
MT	13.95	11.73	7.11	8.08	3.50	9.68
				Grand Mean		9.69

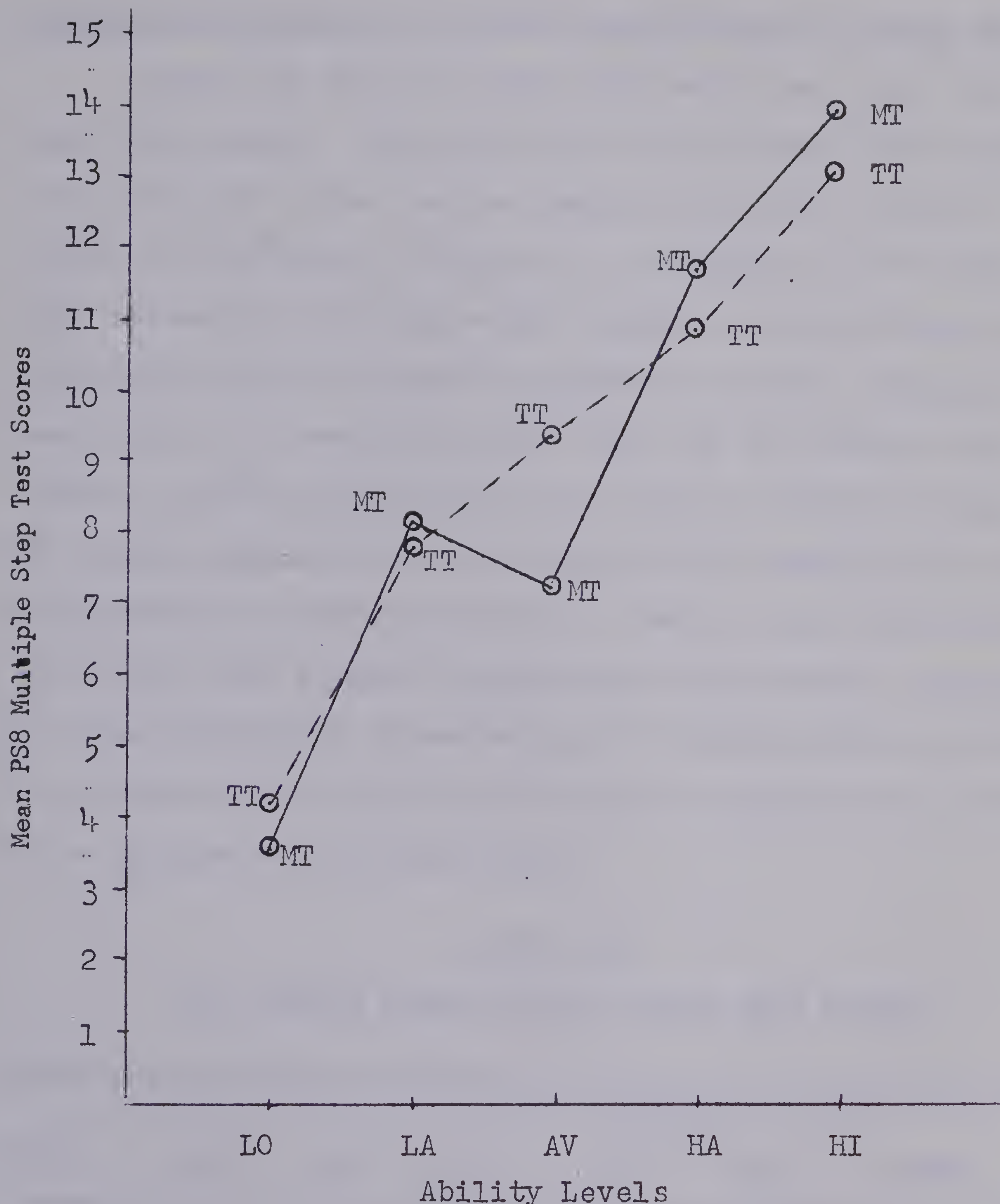
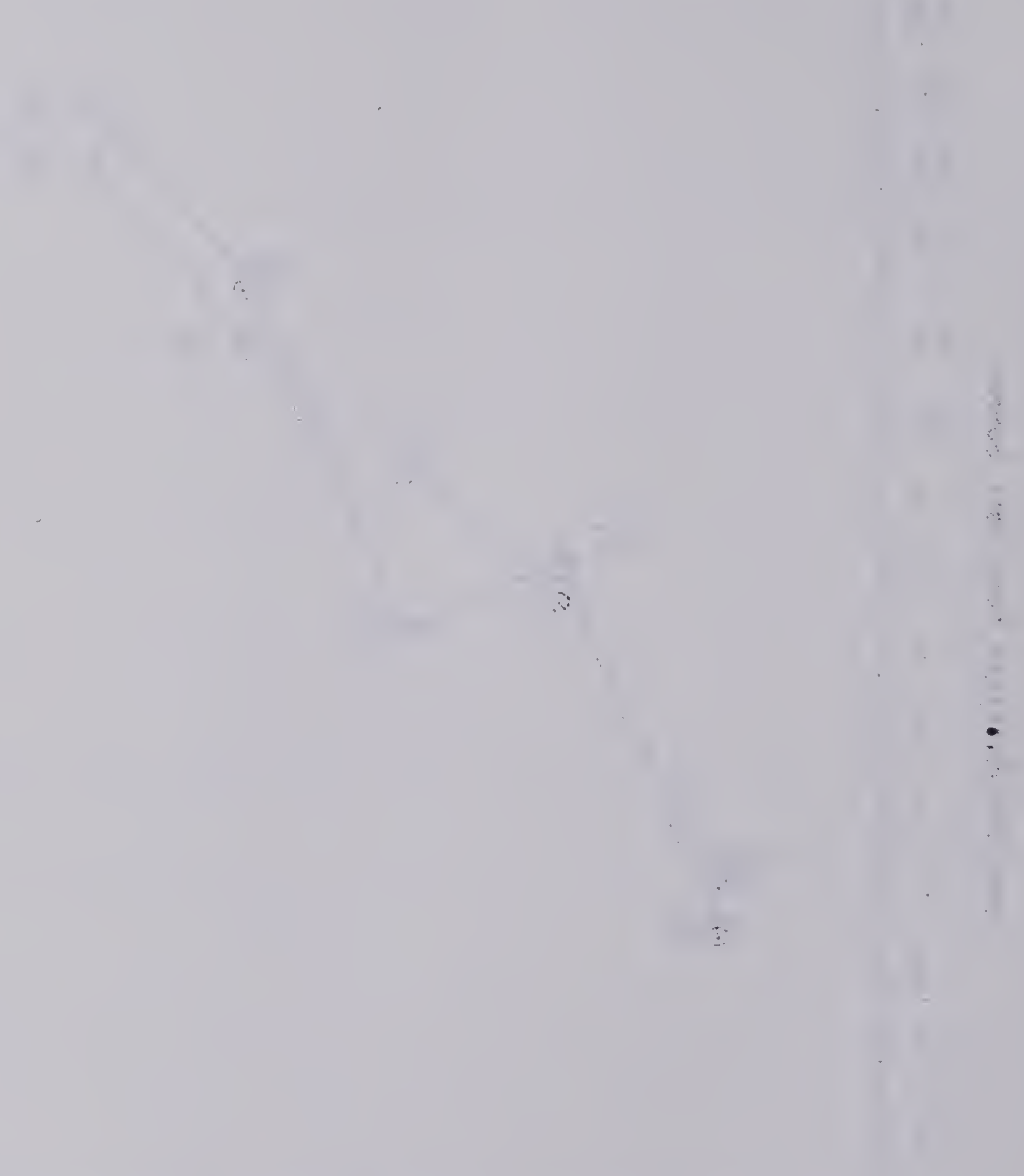


FIGURE 5

CELL MEAN PS8 MULTIPLE STEP SCORES FOR THE TWO GROUPS AT THE FIVE ABILITY LEVELS



Descriptive Analysis of The PS8 Single-Step Test Mean Scores

Table XIV shows the PS8 single-step cell and group mean test scores. The profile of the cell mean PS8 single-step test scores for the two groups at the five ability levels is displayed in Figure 6. The profile of the mean test scores for the single-step section of the PS8 test is definitely different from the previous profiles displayed heretofore. It should be noted that the two ability extremes, the high ability group and the low ability group, HI and LO, appeared to score higher after studying the modern approach to problem-solving. However, the middle ability groups, low average, average and high average, apparently found difficulty in mastering the Gestalt-ratio approach to problem-solving and in particular in applying this method to single-step problem types.

TABLE XIV

PS8 SINGLE STEP CELL AND GROUP MEAN SCORES

Group	Ability					
	HI	HA	AV	LA	LO	GROUP
TT	4.83	4.89	4.56	3.95	2.44	4.45
MT	5.71	4.71	4.06	3.71	3.50	4.44
				Grand Mean		4.44

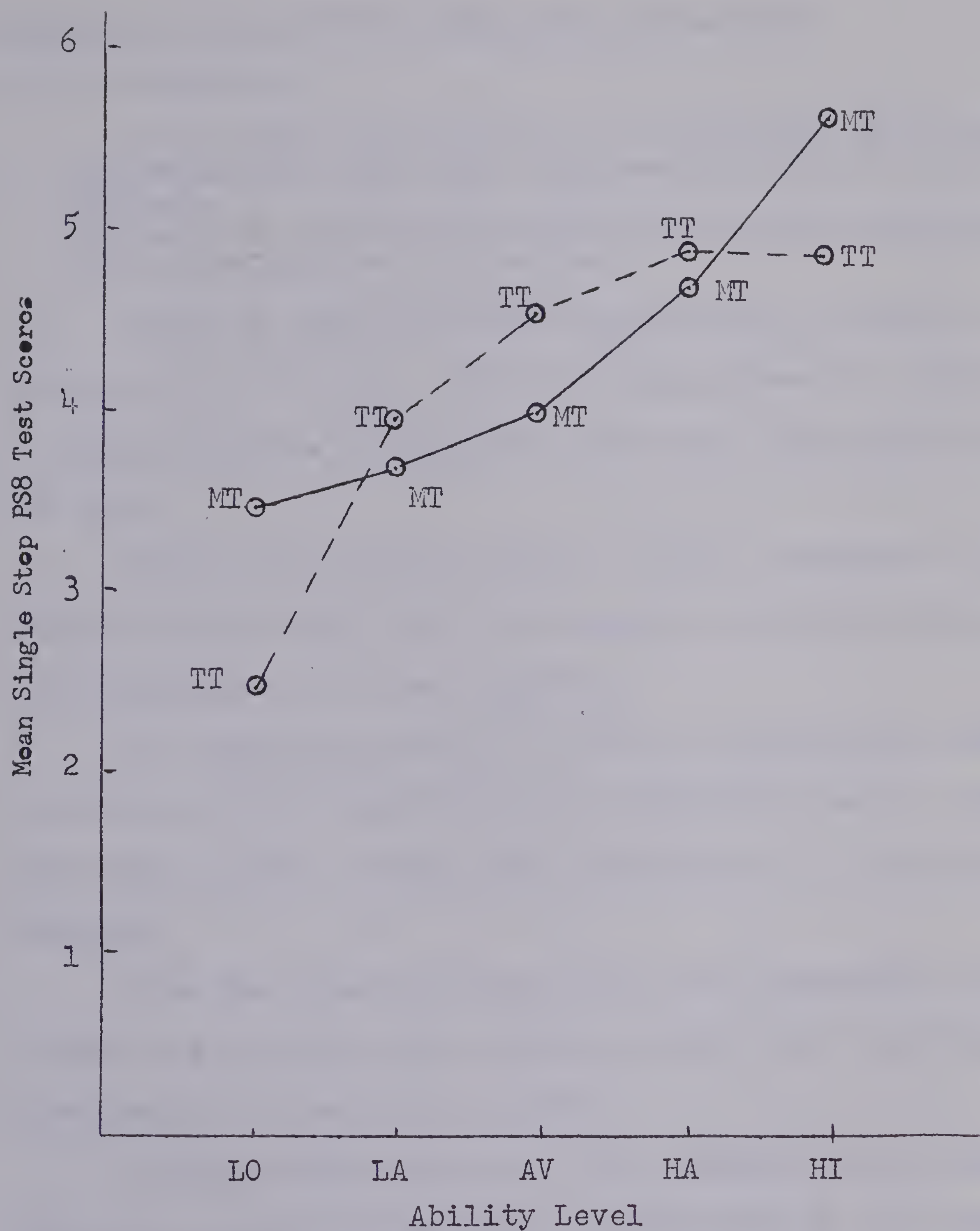


FIGURE 6

CELL MEAN PS8 SINGLE STEP SCORES FOR THE TWO GROUPS AT THE FIVE ABILITY LEVELS

Analysis of The PS8 Test Mean Time Requirement

Null Hypothesis IV

On the PS8 test there are no significant differences
(a) between the mean time requirement of the TT and MT students,
(b) among the ability level cell mean times requirement,
(c) attributable to interaction.

Table XV shows the mean time required by ability groups on the PS8 test. Table XVI summarizes the analysis of variance statistic applied to the mean time scores on the PS8 test.

Since the observed F-ratio (17.879) exceeded the critical ratio (6.75) for the comparison between groups, Null Hypothesis IV a) was rejected.

The observed F-ratio (0.532) for comparisons among the ability level mean time scores was less than the critical value (3.40). Hence, Null Hypothesis IV b) was not rejected.

Also the observed F-ratio for the interaction effect (0.689) was less than the critical ratio (3.40) and Null Hypothesis IV c) was not rejected.

A significant difference was observed between the mean time required to complete the PS8 test by each group. Apparently the MT students worked faster at solving problems with equal proficiency on the PS8 test than the TT students. No significant mean time differences were observed among the ability level groups. No significant interaction effect was

TABLE XV

PS8 CELL AND ABILITY GROUP MEAN TIME SCORES

Group	Ability					
	HI	HA	AV	LA	LO	GROUP
TT	86.44	92.87	88.56	87.00	95.89	89.79
MT	80.43	78.35	80.23	80.50	80.60	79.67
				Grand Mean		84.52

TABLE XVI

SUMMARY OF ANALYSIS OF VARIANCE OF PS8 TEST MEAN TIME SCORES

SOURCE	SS	DF	MS	F
Group	5121.31	1	5121.31	17.879
Ability	609.92	4	152.48	0.532
Interaction	789.52	4	197.38	0.689
Within	73614.81	257	286.44	--

$F_{.01}(1,257) = 6.75$ $F_{.01}(4,257) = 3.40$

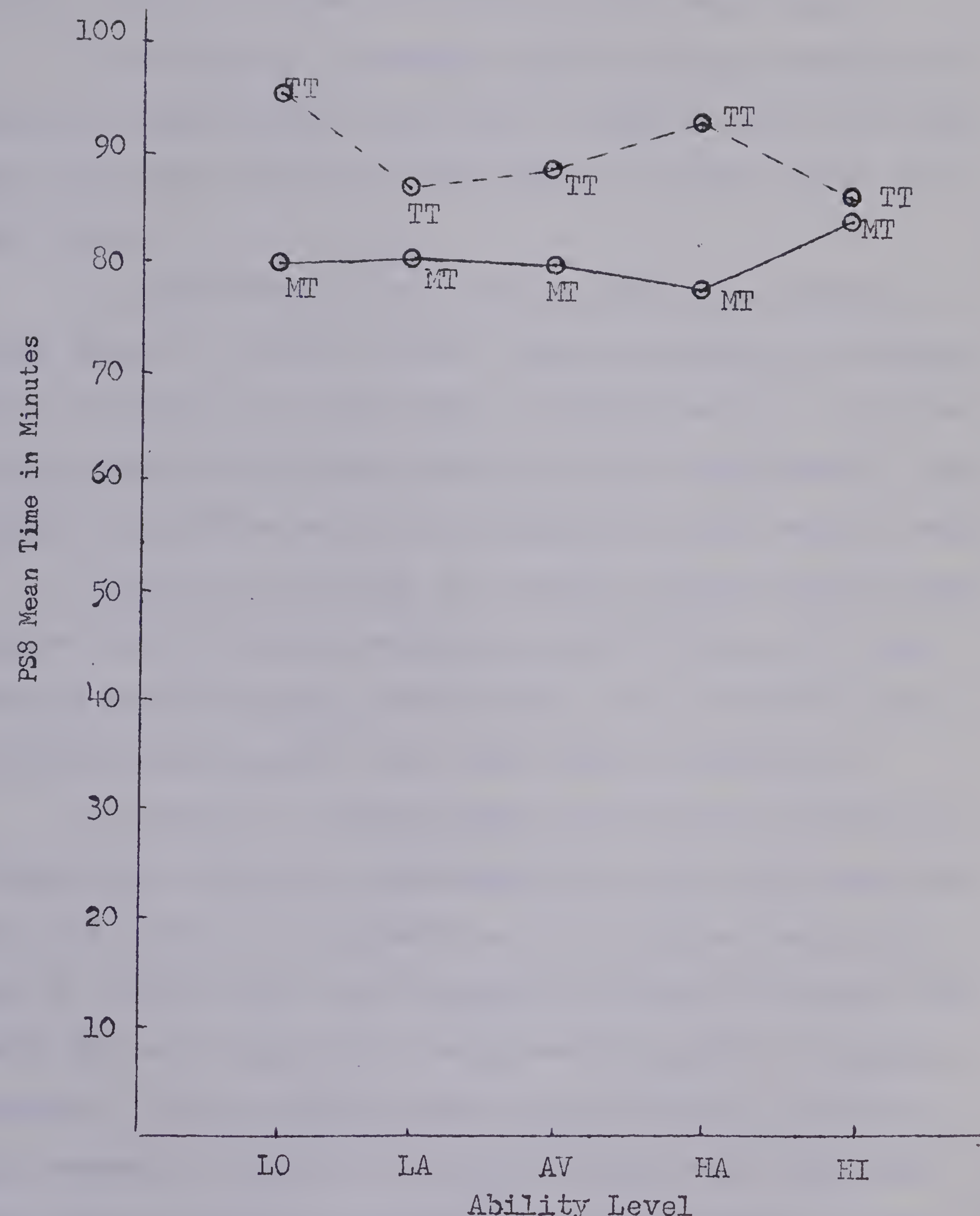


FIGURE 7

CELL MEAN TIME PS8 FOR THE TWO GROUPS
AT THE FIVE ABILITY LEVELS

noted. Figure 7 shows the ability group mean times.

To determine if certain ability groups specifically required significantly less time to complete the PS8 test, the t-test was applied to each pair of ability level cell mean times.

It was observed that the HA group as an ability group showed a t-value (3.545) which exceeded the critical value (1.989) at the .01 level of significance. All other groups showed differences which were not significant, even though the difference at the LO ability level seemed large.

Though the MT group as a whole required significantly less time to solve problems on the PS8 test, the high average ability group, particularly, as an ability group required significantly less time using MT materials.

It should be observed that the F-ratio (0.689) for interaction effect was apparently low which might indicate that the effect of treatments on the required completion time on the PS8 test was decidedly in favour of those students who were taught the Gestalt-ratio method of solving problems. No one ability group apparently was penalized with respect to speed of solving problems when employing this modern approach. Figure 7 seems to indicate a more pronounced ability group variation of mean times in the TT group than the MT group.

Descriptive Analysis of PS8 Test Mean Time Requirement by Achievement Groups[1]

Table XVII represents the PS8 cell and achievement group mean time scores. Figure 8 displays a profile of the cell mean time PS8 scores for the two treatment groups at the three achievement levels.

The profile readily suggests that at all achievement levels, students who learned the modern Gestalt-ratio approach to problem-solving completed the PS8 test in less time than the corresponding achievers who studied the traditional approach. Further, the profile seems to suggest that the better the problem-solver the greater time was required to complete the test no matter to which treatment the students were exposed.

TABLE XVII

PS8 CELL AND ACHIEVEMENT GROUP MEAN TIME SCORES

Group	Achievement			
	PPS	APS	GPS	GROUP
TT	88.06	89.34	91.39	89.79
MT	78.16	79.14	81.42	79.67
			Grand Mean	84.52

[1]See Table VI, p. 54.

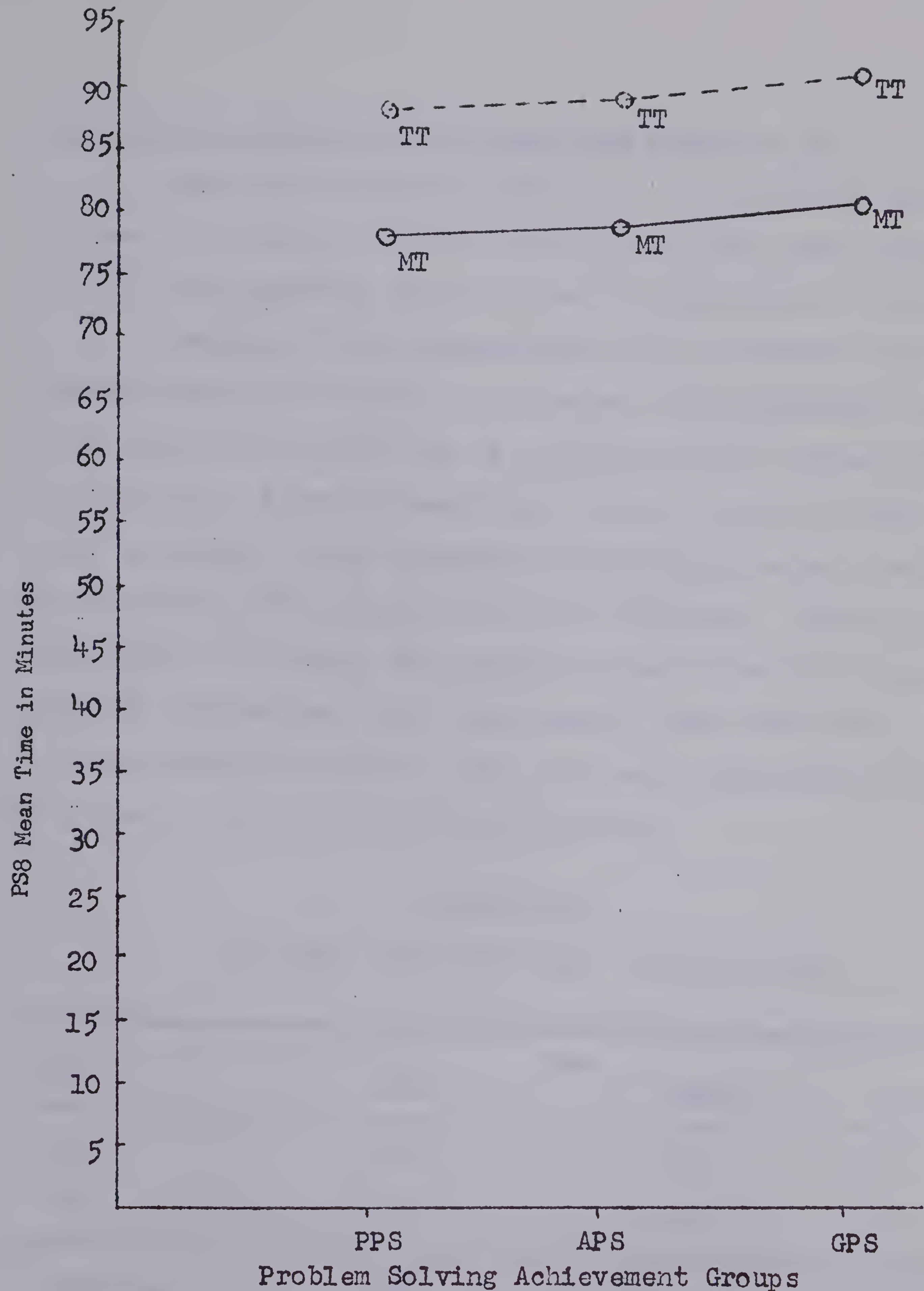
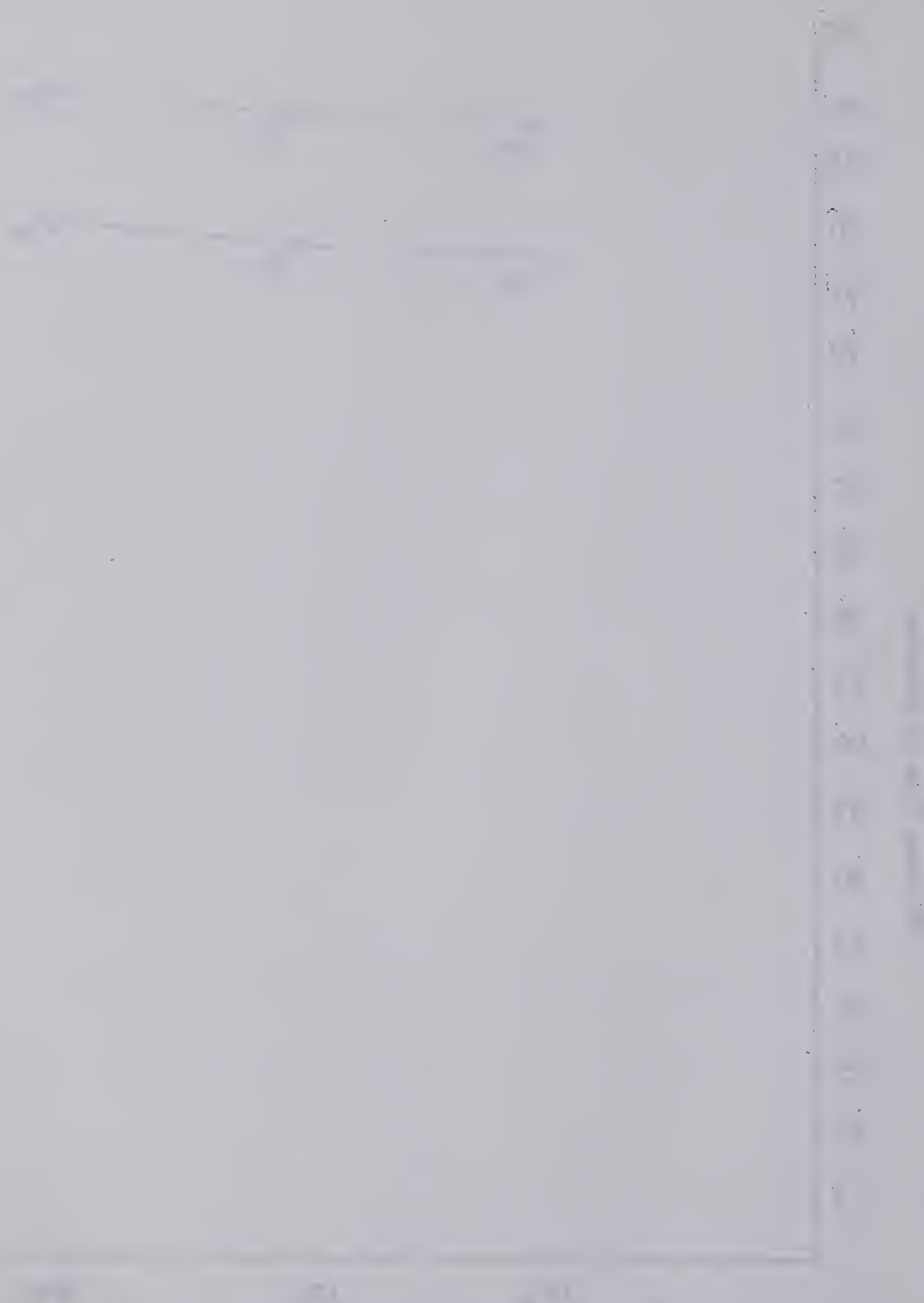


FIGURE 8

CELL MEAN TIME PS8 FOR THE TWO GROUPS
AT THE THREE ACHIEVEMENT LEVELS



<u>Descriptive Analysis of PS8 Mean Test Scores by Sex</u>

Table XVIII shows the PS8 cell and group mean scores by sex. The profile of the PS8 cell and group mean scores for the two treatment groups by sex is presented in Figure 9.

Scrutiny of the accompanying representations seems to indicate that the overall performance, notwithstanding treatment applied, the male sex as a group were the better problem-solvers. Further observation seems to also indicate that no matter which treatment is applied, the male sex as a group seemed to respond better to treatment. However, a suggestion is evident that within the male group the traditional approach may have been somewhat more effective, whereas within the female group the modern approach may have been more effective in solving problems.

TABLE XVIII

PS8 CELL AND GROUP MEAN SCORES BY SEX

Group	Sex MALE	 FEMALE	 GROUP
TT	15.46	13.11	14.15
MT	14.67	13.41	14.10
Combined	15.01	13.25	14.12

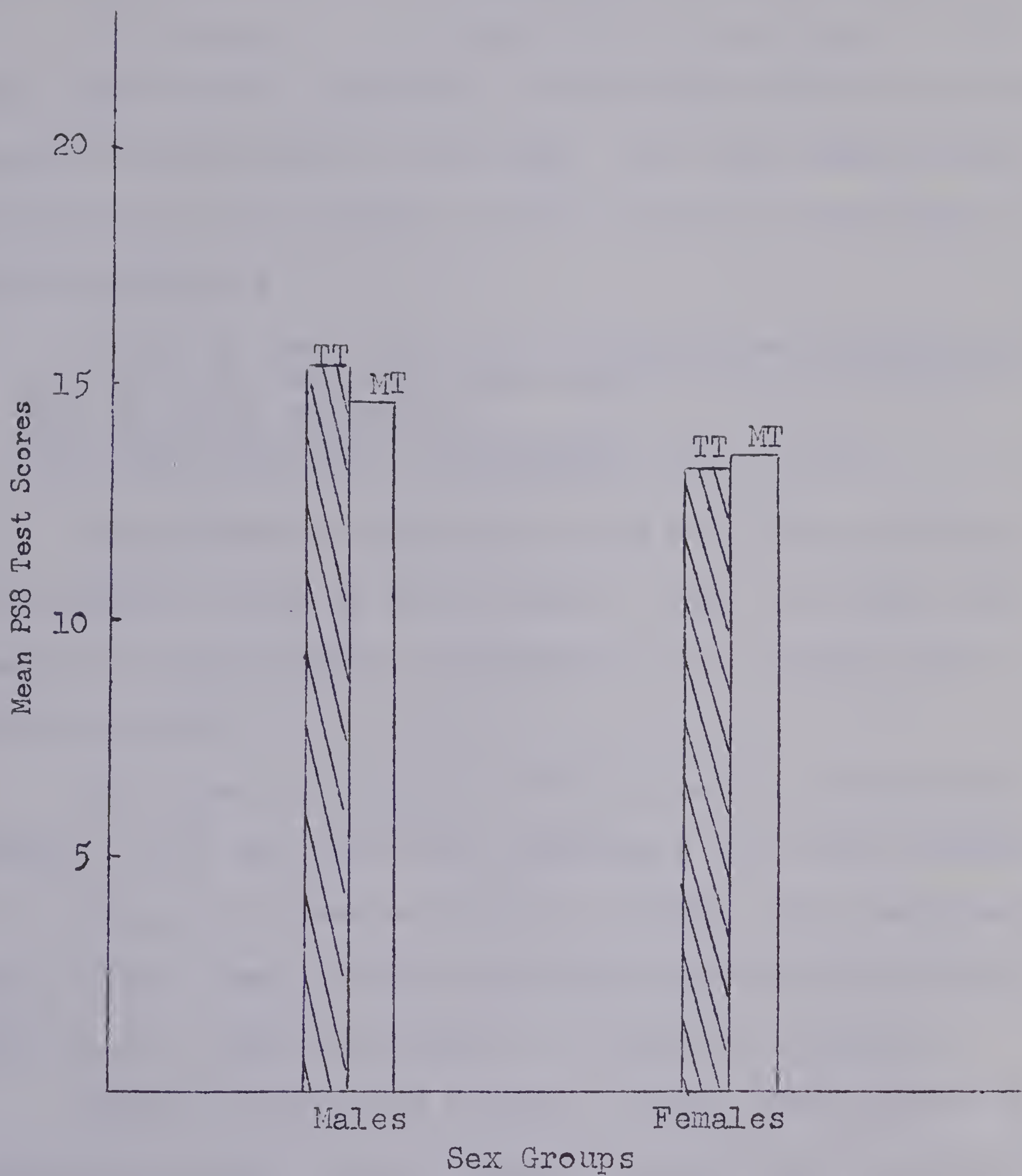


FIGURE 9

CELL MEAN PS8 TEST SCORES FOR THE TWO GROUPS BY SEX GROUPS

Analysis of The M9 Cell and Group Mean Stanine Scores

Of interest to the reader but not pertinent to the study might be the correlation coefficients among the four variables OTIS, PS8, M9, and SCAT. For this reason these calculations were employed and are reported in Appendix E.

Null Hypothesis V

On the M9 test there are no significant differences (a) between the group mean stanine scores attained by the TT and MT students,
(b) among the ability level group cell means,
(c) attributable to interaction.

Inspection of Table XIX reveals the cell and group mean stanine scores on the M9 test. Table XX shows the summary of the analysis of variance of the M9 test mean stanine scores.

The observed F-ratio (7.703) exceeded the critical F-ratio (6.75) and thus Null Hypothesis V a) was rejected.

Since the observed F-ratio (52.236) for comparisons among ability level group cell means exceeded the critical ratio (3.40), Null Hypothesis V b) was also rejected.

Since the observed F-ratio (1.183) attributable to interaction effect did not exceed the critical F-ratio(3.40), Null Hypothesis V c) was not rejected.

A significant difference in achievement between treatment groups was observed. The TT students as a group performed better on the M9 test than the MT students. Also

TABLE XIX

M9 CELL AND GROUP MEAN STANINE SCORES

Group	Ability HI	HA	AV	LA	LO	GROUP
TT	7.44	6.00	5.32	4.23	2.11	5.41
MT	6.76	5.35	4.17	3.25	2.40	4.69
				Grand Mean		5.03

TABLE XX

SUMMARY OF ANALYSIS OF VARIANCE OF M9
TEST MEAN STANINE SCORES

SOURCE	SS	DF	MS	F
Group	20.05	1	20.05	7.703
Ability	543.79	4	135.95	52.236
Interaction	12.32	4	3.08	1.183
Within	668.86	257	2.60	---

$F_{.01}(1,257) = 6.75$ $F_{.01}(4,257) = 3.40$

found was an expected significant difference among ability level groups on the M9 test. The effect of interaction was not significant.

Since a significant F-ratio (7.703) was observed and attributable to effect, and the interaction effect observed was somewhat high but not significant, there was a generalized treatment effect. Nonetheless to further verify this the following hypotheses were tested.

<u>Analysis of M9 group cell means</u>. Null Hypothesis Vb

On the M9 test there is no significant difference
(1) between the HI ability level cell mean stanine scores
(2) between the HA ability level cell mean stanine scores
(3) between the AV ability level cell mean stanine scores
(4) between the LA ability level cell mean stanine scores
(5) between the LO ability level cell mean stanine scores

Figure 10 illustrates graphically for each treatment group the mean stanine scores obtained by each ability level group.

Application of the t-test statistic to pairs indicated that the AV t-ratio (2.739) exceeded the critical t-value (2.644). All other ability level groups obtained t-ratios less than the corresponding critical t-ratios. Therefore, Null Hypothesis V-b-3) was rejected, and Null Hypotheses V-b-1,2,4, and 5 were not rejected.

Thus the average ability group who scored higher on the P88 test did significantly better on the M9 test.

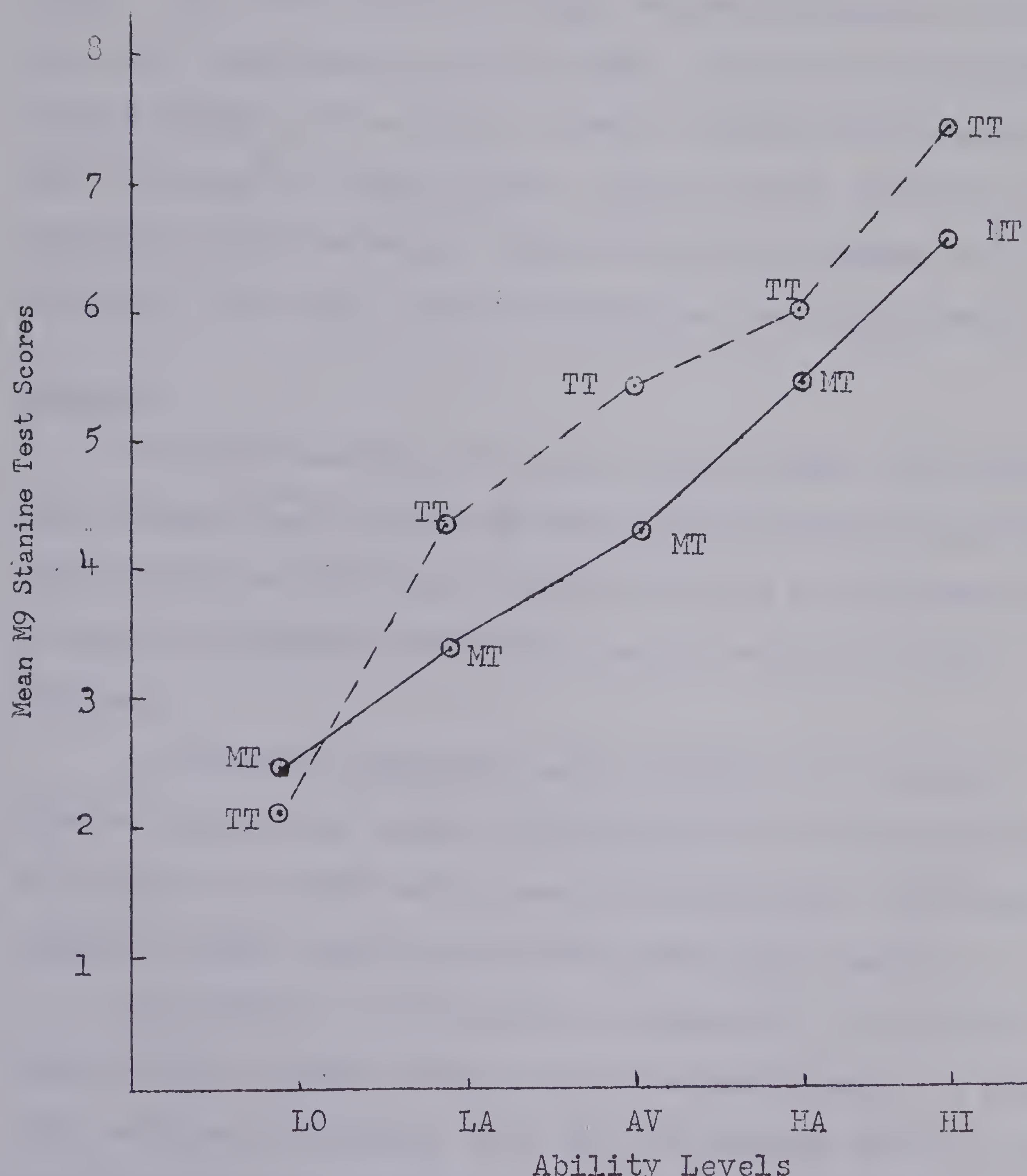


FIGURE 10

CELL MEAN M9 STANINE SCORES FOR THE TWO GROUPS
AT THE FIVE ABILITY LEVELS

The difference in means was again significant at the .01 level. All other ability groups scored differences which were not significant at this level. Both HA and LA ability groups studying TT materials scored significantly better at the .05 level of significance. The LO group using the MT approach scored somewhat better than the LO group in TT materials but again the difference was not significant.

Summary

In the treatment of data in this study, the difference between means was considered statistically significant only if the probability of observing such a difference as a result of sampling error was one or less out of one hundred.

A two-way unweighted means analysis of variance carried out on the scores obtained by the TT group and the MT group on the PS8 test showed no significant difference existed between the two treatment group mean scores.

Significant differences, as expected, occurred between ability groups within each treatment group. A phenomenon occurred, it seemed, when the low average ability students studying MT materials scored higher than the average ability students studying the same materials.

It appeared that the higher ability students studying modern materials as well as the lower ability students in the

same treatment group scored better than the corresponding students in the traditional treatment group. The profile also seemed to suggest that the average ability student may have encountered difficulty in mastering the modern approach to problem-solving.

On the rate problem test section no significant difference between the two treatment group mean test scores was observed. The expected significant difference among the ability level rate test mean scores was observed. The phenomenon of the unexpected difference in mean scores between the low average and average ability groups of the modern treatment group seemed to be evident.

Inspection of the profile of rate problem test question ability group mean scores seemed to suggest that the above average ability students who studied the modern materials may have done better than the corresponding ability group studying the traditional materials. This same pattern apparently existed for the low ability student. The applied treatments seemed to have the opposite effect on the average ability student, however.

On the PS8 non-rate problem types no significant difference between treatment group mean scores was observed. The expected significant difference among the ability level mean scores was observed. The interaction effect remained not significant. However, the same phenomenon between the

low average and average ability group in the modern treatment group seemed to exist. The pattern of apparent differences in mean scores of each treatment group for each ability level was noticeable on the profile of mean test scores of non-rate problem types with the one exception, that the low ability group in the modern treatment group appeared to have scored lower than the corresponding ability group in the traditional treatment group.

Analysis of the multiple-step problem types seemed to suggest that the above average ability students studying the modern materials may have responded more satisfactorily to this treatment. However, the average ability student once again seemed to respond better to the traditional approach to problem-solving. Whatever apparent difference in means was observed in the low ability group seemed to favour the modern treatment group. The phenomenon of the unexpected difference in means of the low average and the average ability students in the Gestalt-ratio treatment group appeared to be evident.

The descriptive analysis of the single-step test mean scores seemed to indicate a rather different profile than what was previously reported. The two extremities of ability levels seemed to have responded more favourably to the modern approach to problem-solving. However, the middle ability groups seemed to respond better to the traditional treatment.

On the PS8 test mean time scores analysis it was noted that a significant difference existed between treatment group mean time scores. Comparisons among ability level groups indicated no significant difference existed. The effect of interaction was not significant. When the t-test was applied to ability level mean time scores by treatment group, it was noted that the high average ability group showed a significant mean difference in favour of the MT treatment group. Nevertheless, all ability groups apparently benefited from the modern treatment with respect to solving time. Examination of the profile of mean time scores seemed to suggest a more pronounced variation in completion times within the traditional group.

The descriptive analysis of the PS8 test mean time requirement by achievement groups seemed to suggest as expected that at all achievement levels the MT treatment group required less time to complete the PS8 test. The profile further suggested that the better the problem-solver the greater length of time seemed to be necessary to complete the test no matter which treatment was used.

Upon investigating statistics of the sex groups in relation to achievement, there is a suggestion that the male sex may have been the better problem-solvers. However, the profile seemed to indicate that the traditional treatment

may have benefited the male sex, whereas the female sex seemed to respond more favourably to the modern treatment.

Analysis of the M9 test mean scores indicated that a significant difference existed between treatment group mean scores in favour of the traditionally taught group. As expected, though, a significant difference existed among ability level groups within each treatment group. The effect of interaction was not significant.

A further analysis of ability cell mean scores showed a significant difference between the average ability level students in favour of the traditional group. The same phenomenon seemed to be present in the PS8 test mean scores as well. All other apparent differences were in favour of the ability groups studying the traditional approach with the one exception of the low ability group.

CHAPTER V

SUMMARY, CONCLUSIONS, LIMITATIONS AND IMPLICATIONS FOR FURTHER RESEARCH

Summary

The purpose of this study was to evaluate a plausible and unsophisticated but careful statistical method of comparing two methods of teaching problem-solving at the junior high school level. The one approach, TT, was traditionally oriented to the _Study Arithmetic_ and _Winston Mathematics_ textbook series. The alternate approach, MT, was considered modern and was oriented to the _Seeing Through Arithmetic_ series and its sequent series, _Seeing Through Mathematics_, but employed a modified _Winston Mathematics_ program using _Seeing Through Arithmetic_ concepts of mathematics in the junior high school.

Six classes totalling 128 students, designated the control group, TT, were administered the _Otis Test of Mental Ability, Beta, Form EM_, (OTIS) and the _Problem Solving Eight_ test, (PS8), in their grade eight year, June, 1963. Another six classes totalling 139 students, MT, from the same schools the following year, June, 1964 were administered the same two tests. In the two succeeding years the two experimental groups completed grade nine and wrote the _Cooperative School and College Ability Test, Level Four_, (SCAT), and the

grade nine mathematics departmental examination (M9), but both groups studied the traditional Mathematics for Canadians, Book 1 in their respective grade nine years.

Ability groups, LO, LA, AV, HA, HI, were defined using OTIS scores; time level groups, SW, AW, and FW were established from the time scores on the PS8 test; and achievement groups, PPS, APS, and GPS, were determined from the achievement scores on the PS8 test.

Null hypotheses[1] were formulated from the questions[2] upon which the study was designed. The null hypotheses were subjected to the test of analysis of variance and appropriate t-tests were employed, when applicable, to individual pairs of subgroups. Descriptive analysis of various subtests and variables was prepared and presented accordingly.

Conclusions

The conclusions which apply to the twelve classes of 267 grade eight students in the County of Beaver #9 are as follows:

I. On a special problem-solving test at the end of grade eight

(a) students studying a traditional approach to problem-solving as a group did equally well as students studying the Gestalt-ratio approach to problem-solving,

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[1] See page 6

[2] See page 40

(b) significant differences among the ability level groups within each treatment group were observed as expected,

(c) the effect of interaction between the variables of ability and treatment was not significant.

II. On a special problem-solving test at the end of grade eight, scores on the rate problem section showed that:

(a) students studying traditional materials did as well as students studying the modern materials,

(b) significant differences among the ability level groups within each treatment group were observed as expected,

(c) the effect of interaction between the ability and treatment variable was not significant.

III. On a special problem-solving test at the end of grade eight, scores on the non-rate problem section showed that:

(a) students studying the modern approach to problem-solving scored as well as the students who studied the traditional approach,

(b) significant differences among the ability level groups within each treatment group were observed as expected,

(c) the effect of interaction between the ability and treatment variable was not significant.

IV. On a special problem-solving test at the end of grade eight, the student-recorded time to complete the test showed that:

(a) a significant difference in completion time existed in favour of the students studying the modern approach to problem-solving,

(b) no significant difference was observed among ability level groups with respect to completion time,

(c) no significant difference was noted between the effects of ability and completion time.

V. On the M9 mathematics departmental examination at the end of grade nine, evidence showed that:

(a) students using a traditional approach to problem-solving as a group scored significantly higher than the students studying the Gestalt-ratio approach to problem-solving,

(b) significant differences existed among the ability levels within each treatment group as expected,

(c) the interaction effect between the variables of ability and treatment was not significant.

Assuming the validity of the PS8 test and assuming that the initial problem-solving proficiency of the two treatment groups was not significantly different, one might conclude from the study that the Gestalt-ratio approach to problem-solving was no more effective than the traditional approach in effecting problem-solving proficiency. As expected each ability group within each treatment group responded according to their ability level. The effect of interaction of the controlled variables of treatment and ability did not seem to be significant in the study.

The Gestalt-ratio group of students studied an approach which emphasized the employment of ratio application

to problem-solving whenever possible. Accordingly then, the MT group might be expected to perform better than the traditional, TT, group in solving rate problem types. However, students in both treatment groups did equally well. The effect of interaction between the variables of treatment and ability seemed to make little contribution to the pattern of results as disclosed through mathematical analysis.

The same pattern of results previously observed seemed also evident in the non-rate problem types. Both treatment groups did equally well with this type of problem situation. The effects of ability and treatment seemed to still show little effect on the results of the study.

Nevertheless, it was certainly evident from the study that the time required to complete the PS8 test according to instructions given to the student was significant. Students who studied the Gestalt-ratio approach to problem-solving benefited from the treatment since this group required significantly less time to complete the test. Further, observations indicated that in either treatment group no significance may be attached to completion time and ability level. No one ability group required significantly greater or less time to complete the PS8 test than any other ability group. It was noted, however, that between ability groups, the high average ability group as a group studying modern materials required significantly less time than the high average ability

group studying the traditional materials.

On the grade nine departmental examination in mathematics the analysis of results gave evidence to show that the students who studied the Gestalt-ratio approach to problem-solving in the few years prior to entering grade nine did not appear to benefit on this part of the test as a result of the modern treatment. In fact it appeared that the students of the modern treatment group may have suffered on the possibly rather traditionally oriented grade nine mathematics examination.

On the PS8 test a seemingly unusual phenomenon occurred in the total test score as well as the rate and non-rate sections of the test. In the modern treatment group the low average ability students seemed to score higher than the next higher ability group, the average ability students. It also appeared that the higher ability students studying modern materials as well as the lower ability students scored better than their respective ability groups who studied the traditional materials. The average student, as found in this particular sample, seemed to have had some noticeable difficulty in mastering the modern approach to problem-solving.

The profile of the mean rate test scores according to ability seemed to suggest that except for the average ability students, all other ability groups may have benefited

from instruction in the modern approach to problem-solving.

The profile of the multiple-step problem types did not appear to be much different from that of the non-rate test scores and the total PS8 test score means.

However, the profile of the single-step mean test scores showed a rather different pattern from the previously noted profiles. Ability groups at the two extremities, the low ability students and the high ability students seemed to respond better to the modern treatment; whereas, the middle ability groups seemed to react apparently opposite and seemed to respond better to the traditional approach to problem-solving.

Scrutiny of the profile of mean PS8 time scores seemed to suggest a more pronounced variation in completion times within the traditional treatment group. In comparing completion time scores by achievement groups, it seemed that the better the problem-solver the greater the length of time was necessary to complete the test no matter which treatment was apparently used.

In looking at the profile of mean test scores by sex of student it would appear that the male sex appeared to be better problem-solvers. Indications seemed to suggest that the male sex responded possibly better to the traditional treatment; whereas the female sex may have responded better

to the modern treatment.

One might conclude from the study that the analysis of data technique as presented in the investigation may be reasonably employed assessing the effects of changing mathematical programs with reasonable accuracy and apparent success in the small rural school jurisdiction. The analysis of data as presented in this study would seem to exhibit a level of sophistication which might be employed in similar rural school authorities who might find some difficulty in meeting some of the more rigid assumptions of statistical analysis in education and yet be descriptive enough to enable conclusions to be formulated about the perticular population and the resultant effects of treatments applied. The study of profiles of mean scores compared against another variable presents a rather reasonable picture of the apparent effects of treatments one may wish to evaluate. Such descriptive statistics apparently do not require considerable involved labour and might well be suited to the kinds of requirements found in rural school jurisdictions. Of course the reader must accept certain limitations which are inherent in the more simple studies applicable to the local school situation and must be aware

of the tenuous nature of strong conclusions reached under these circumstances.

Limitations

The investigator acknowledges some obvious limitations that must be placed on the interpretation of the findings of this study. First, the participants of the study were not a random sample of the Alberta population but only a segment of a local population. A second limitation might be the lack of a relative estimate of problem-solving proficiency and knowledge of mathematical understandings at the end of grade five before the new texts were employed. Though both groups were of apparently equal proficiency in problem-solving at the end of grade eight, the investigation can not state that improvement did or did not appear over the three year study. A third limitation might be the appropriateness of the special problem-solving test produced by the investigator for the study. A fourth limitation would involve the fact that a time-lapse approach employing two different groups would create further restrictions. A final limitation might be the degree of validity of the conclusions that can be based on the M9 test and corresponding stanine scores. The

actual distributions of raw scores within each stanine is not known but can only be assumed to be the same for both groups.

Implications and Recommendations

Certainly the most apparent implication for future research in the field of mathematics is that the experimental study reported herein should be replicated with tighter experimental controls. Since modern textbooks were authorized at the grade seven level in Alberta schools in September, 1965 and in each next higher grade in subsequent years, the possibility of extracting a random sample from the Alberta junior high school population would ensure a very important control factor of the experiment.

The claim of textbook superiority of the authors and publishers of the Seeing Through Arithmetic and Seeing Through Mathematics series should be researched more definitively, particularly in the realm of problem-solving. Specifically the Gestalt-ratio approach as a tool to particularly multiple-step and ratio problem-solving success should be investigated.

Further investigation into problem-solving proficiency by various treatments, but particularly the Gestalt-ratio treatment, in relation to effective responsiveness of ability groups would be useful. The present study seemed to

suggest that students of 85 I.Q. and lower generally appeared to do better on most sections of problem-solving as well as general mathematics achievement when exposed to the Gestalt-ratio approach. The same effect was suggested at the I.Q. level of 105 and above. The phenomenon of the average ability student's apparent ineffective response to modern approaches should be investigated.

The economy of time to set up a problem and find an answer was found significant in the study. However, the Gestalt-ratio tool as a computational device should be thoroughly investigated apart from the general organization of solutions in solving problems.

While the study reported herein suggested that a possibility existed that considerable transfer of knowledge may have been effected between grade eight and grade nine, the whole area of transfer of principles, ideals and attitudes as proposed by Bruner[3] and Bloom[4] should be investigated in relation to various approaches to the teaching of mathematics.

The limitation of the effect of transplanting a new and different approach upon an already established approach

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[3]Jerome S. Bruner, The Process of Education, (Cambridge: Harvard University Press, 1962).

[4]Benjamin Bloom, Taxonomy of Educational Objectives, (New York: Longmans, Green, 1956).

should be eliminated. A longitudinal study commencing at kindergarten or grade one and extending for a number of years with periodic evaluations would do much to provide empirical evidence of the effectiveness of new mathematical programs.

Certainly if ability groups tend to respond differently to various kinds of treatment as suggested in this study, a definite implication might be further studied. It may be administratively and educationally sound to group students according to both ability and treatment for the purposes of more effective and efficient instruction. This implication should be more thoroughly investigated not only in mathematics curriculum but possibly also in many other subject areas.

A further implication might suggest the advisability of multiple authorizations of textbooks by the provincial department of education in possibly all curriculum areas. If the pattern of varying achievement by ability seems to be evident, a need for textbooks which are oriented to different approaches might be useful teaching devices in the classrooms of the nation.

Implied in this study is the necessity of researching the literature and previous investigations in relation to the problem before implementing any change in educational programs. Such review and research will undoubtedly assist the educator in the preparation and execution of new programs.

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APPENDIX A

INSTRUCTIONS GRADE EIGHT PROBLEM SOLVING TEST

A. PRELIMINARY:

(1) Distribute face down one paper to each student. Upon completion of this phases at the command students will turn paper over for further instructions.

(2) Assist pupils in completing the required information regarding name, teacher, and so on.

(3) Read carefully the instructions on the front page.

(4) Read carefully and do the sample problems, noting the following:

(a) Space for Statements or Equations -- Students must write all statements or equations concerning the process of thinking in processing the final answer.

(b) Calculations must be done in the space provided and on the back side of paper if necessary.

(c) Insert the answer in the space provided.

(d) Each question is blocked. Try to stay within the block or on the backside of the block.

(5) Answer questions by students concerning procedure. No questions concerning interpretation or vocabulary is allowed during the testing period.

B. TESTING:

(1) There is no time limit on this test. Provide enough time so that everyone completes or does what they can. Two periods will probably be sufficient.

(2) In the space marked time insert the number of MINUTES (to the nearest minute) that the student required to complete the test to the best of his ability including a review if necessary. TIME IS A MOST IMPORTANT FACTOR.

(3) All work is to be done in the test booklet. No extra paper is required.

(4) Write neatly in PENCIL.

(5) The diagram for problem 20 and sequent problems is probably not clearly printed. Draw an outline on the blackboard providing these dimensions: City- 18' x $12\frac{1}{2}$' Farm- $16\frac{1}{2}$' x 14'.

(6) Collect all papers as completed and submit the bundle to the principal for disposition.

:::::::::::::::::::::::::::::::::::

PROBLEM SOLVING TEST

GRADE 8

TIME __________

NUMBER RIGHT __________

NAME________________TEACHER________________SCHOOL_______

BOY______GIRL_______GRADE_________DATE_______________

BORN___________________________AGE_______________________

month day year month day years

DIRECTIONS: Read each problem carefully and be sure to do just what it asks. Use the space "statements or equations" to set up the problem as shown in the example. Do your rough work such as calculations in the space "calculations". Be sure to write your answer on the dotted line at the side of the page. Note the sample problems.

SAMPLE PROBLEMS: The pupils in one room changes the soil for plants in their room. They had two sizes of flower pots. The large pots each held two quarts of soil and the small pots each held one quart of soil.

<u>The sample question worked for the student utilizing the (TT) approach to problem-solving.</u>

A. How many quarts of soil were needed for 7 of the large pots?

Statements or Equations	Calculations	Answer
There are 7 large-sized pots	$\begin{array}{r} 7 \\ \underline{\times 2} \\ 14 \end{array}$	A.....14..qt.
Each pot holds 2 quarts		
The number of quarts will be 7x2		

B. For new soil for the plants 1 quart of sand was used for each 2 quarts of soil from the garden. How much sand was used with a bushel (32 quarts) of soil from the garden?

Statements or Equations	Calculations	Answer
1 quart of sand is used for each	$2\overline{)32}$ = 16	B.....16..qt.
2 quarts of soil.		
32 quarts of soil were used.		
At that rate the number of quarts used was 32 divided by 2		

The sample question worked for the student utilizing the (MT) approach to problem-solving

A. How many quarts of soil were needed for 7 of the large pots?

Statements or Equations	Calculations	Answer
$\frac{1}{2} = \frac{7}{n}$	$\begin{array}{r} 7 \\ \times 2 \\ \hline 14 \end{array}$	A. 14 qt.
$n = 2 \times 7 = 14$		
14 quarts of soil were needed		

B. For new soil for the plants 1 quart of sand was used for each 2 quarts of soil from the garden. How much sand was used with a bushel (32 quarts) of soil from the garden?

Statements or Equations	Calculations	Answer
$\frac{1}{2} = \frac{n}{32}$	$2\overline{)32}$ = 16	B. 16 qt.
$2n = 32$		
$n = 16$		
16 quarts of soil were used.		

VACATION ON THE FARM

George and Alice Peters live in the city. They like to spend summers with their cousins Lucille and Larry on the Farm. Sometimes Lucille and Larry visit in the city. The train fare between the city and the farm is $5.88 each way for a grown person and half as much for a child under 12 years of age.

1. Alice is 11 years old. How much is the train fare for her from the city to the farm?

Statements or Equations	Calculations	Answer
		1.$........

2. Mr. and Mrs. Peters and George use full-fare tickets. How much do these tickets cost for the trip to the farm for the 3 people?

Statements or Equations	Calculations	Answer
		2.$.......

3. One summer Mr. Peters took the children is his car. When they started, the speedometer read 49,826 miles. When they arrived, it read 50,031 miles. How far did they drive?

4. They started at 8:30 in the morning and arrived at 10 minutes after 3 in the afternoon. How much time was used in travelling?

5. When Mr. Peters had driven 250 miles, he had used 13 gallons of gas. To the nearest whole mile, how many miles was this per gallon?

6. For spending money, George received 75¢ per week and Alice 50¢ per week. What was the total for six weeks for both children?

7. The children rode Larry's pony. It could run a mile in 5 minutes. How many miles per hour is this?

8. Alice helped her cousin make curtains. Each curtain was 60 inches long and 6 extra inches per curtain were used in making hems. How many yards of material were needed for six curtains?

9. On the farm, the children got up at 6:45 in the morning. When did they go to sleep if they had $9\frac{1}{2}$ hours of sleep?

10. They served fried chicken to a group of friends on Sunday. Three chickens each weighing $3\frac{1}{2}$ pounds were used. How many people did these serve if $\frac{3}{4}$ of a pound was allowed for each person?

''''''''''''''''''''''''''''''''''''

CHICKENS ON THE FARM

The farm has a few cattle and raises some grain for feed, but specializes in raising chickens and selling eggs. Eggs that are fresh, clean, and large bring the best prices. Alice and George learned to sort and grade the eggs.

11. Medium-sized eggs usually sell for 80% as much as large eggs. When large eggs are 55¢ per dozen, for how much should medium-sized eggs sell?

12. Large eggs weigh 28 ounces per dozen and extra-large eggs weigh 32 ounces per dozen. The large eggs weigh what per cent as much as the extra-large eggs?

13. At the right is a chart showing the percentage of content of some chicken feed. What percent of the content of this feed is not accounted for on this chart?

Feed Content	
Protein.......	20%
Fat...........	4%
Fiber.........	8%

14. Larry and George brought home a load of feed that weighed 2250 pounds. How much did this feed cost at $2.80 per hundred pounds?

15. In preparing feed for laying hens, Larry had to put in $\frac{1}{2}$ of 1% salt. How many pounds of salt did he use in mixing 1500 pounds of feed?

16. Mrs. Peters pays 54¢ per dozen for eggs for which the farmer gets 45¢ per dozen. Her cost is what percent more than the price the farmer gets?

17. To build another chicken house, Larry's father borrowed $1600. If he paid it back in four months, how much was the interest? The interest rate was 4% per year.

18. In a new breed of chicken, 60% of the weight is edible. In a 5-pound chicken of this breed, how much is inedible?

19. To produce one pound of chicken, 4 pounds of feed are needed. Feed costs $3\frac{1}{2}$ cents per pound. If the chicken sells for 35¢ per pound, what per cent of the selling price is spent on feed?

THE CITY AND THE FARM

In the city George and Alice spend much of their spare time reading. On the farm, they helped with the work. George helped his cousin with the outdoor work and Lucille helped in the house. It was a large pleasant house.

City Home 18 feet by $12\frac{1}{2}$ feet   Farm Home $16\frac{1}{2}$ feet by 14.

20. Two living room floors are sketched above. Which is larger, and how many more square feet of space does it contain? (If they are both the same in size, write same.)

21. The perimeter of the city living room is how many feet more than the perimeter of the farm living room, or are both the same?

22. In the city, people cook with gas. They pay $12\frac{1}{2}$ cents per 100 cubic feet of gas. How much is the bill for a month when 1700 cubic feet are used?

23. The farm uses electricity for cooking, washing and other machinery. The rate is $5 for all the electricity up to 100 kilowatts, and $3\frac{1}{2}$ cents per kilowatt for any extra beyond the first 100. How much is the bill when 238 kilowatts are used?

24. The round water tank on the farm is 6 feet in diameter and 2 feet, 4 inches deep. How many cubic feet of water will it hold? ($V = \pi r^2 h$)

25. One day the boys made a rectangular yard for chickens. They used 120 yards of fence to enclose it. If the chicken yard was two times as long as it was wide, how long was it?

26. If Alice lives "10 minutes by bus" from her school and the bus averages 10 miles per hour, how many miles is it to the school?

27. For his science class, George recorded the following low temperatures one week: -1, +6, +10, -2, -8, 0, +7,. What was the average of these temperatures?

END OF TEST. LOOK OVER YOUR WORK   NO. RIGHT......

APPENDIX B

COUNTY OF BEAVER NO. 9

A GUIDE FOR A MODIFIED PROGRAM IN GRADE SEVEN MATHEMATICS FOR USE WITH PRESENT WINSTON MATHEMATICS

BOOK 1

1962 - 63

Written and Compiled by:

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Supervisor of Instruction

A MODIFIED PROGRAM IN MATHEMATICS FOR GRADE 7 1962 - 63

1. PURPOSES In the school year 1961 - 62 several classes of grade six have been taught the Scott-Foresman (Gage) Seeing Through Arithmetic. These several classes were rather uniquely formed in that they consisted of the better ability students.

It is therefore quite evident that a modified arithmetic or mathematics program is required for the 1962 - 63 school year for the following reasons:

(1) The present grade 7 mathematics cousse as authorized in Winston Math Bk. 1 is now in parts not applicable following the Gage program in grades 1 - 6.

(2) Being that these classes are generally of better than average ability a extensive and enriched program should be devised for their purpose in grade 7 , Septemebr 1962.

(3) The new approach (Gestalt) to problem solving as presented in the Gage program should be continued and not dropped. We feel that this approach will solve many of our problems in mathematics in junior high school especially that of problem solving.

(4) With the new approach, also, we feel that generally better mathematics students should be made if the general concepts of the Gage program in 1 - 6 are continued into junior high school and eventually senior high school.

(5) Since the sub-committee on junior high school mathematics has made no concrete guide for students who have completed the grade 6 Gage year, it is absolutely necessary in our system to do so at this time. This will serve as this guide until the sub committee formulates one.

2. PROBLEMS:

A. PROBLEM SOLVING:

Students with a background from Study Arithmetic or the Ginn series of Arithmetic We Need have been taught to keep certain questions in mind when examining a problem. (e.g.) What am I asked to find? Should I add, subtract, multiply or divide? Also in Ginn the "Cue Word" concept is taught in the methods of problem solving. However, our immediate problem concerns the new approach to problem solving as presented in Gage.

The Gage series spends much time presenting an "equation approach" to problem solving. It is suggested that students utilize a four-step procedure as follows:

1. Read the problem, analyze the situation and note the action that is taking place.
2. Express the situation in the form of a mathematical sentence or equation.
3. Compute to the side.
4. Answer the question asked in the problem

Sometimes a check (substitution) is placed following the computation and before tje final statement of the answer is written.

For example, the solutiom to the following problem might be as indicated:

"Joan bought 2 packages of notebook paper at 37¢ each, and a package of pencils for 49¢. How much did she spend for all these things?

a + \$.49 = d	\$.37	\$.74
(2 x \$.37) + \$.49 = d	2	\$.39
\$.74 + \$.49 = d		
\$.74 + \$.49 = \$1.13	\$.74	\$1.13

Joan spent \$1.13 for all these things

References: S.T.A. - Grade 6, page 50
S.T.A. - Grade 6, Teacher's Guide page 67

Note: students are encouraged to have the equation express the EXACT and ENTIRE situatim in the problem and verbal statement other than the concluding one are discouraged. Few rules are given for finding the letter used in the equation.

B. RATIO

This term has been scarcely referred to in Study Arithmetics. In the Ginn series the term is referred to and defined as a relation between two numbers. In the Gage series much time is spent developing the concept of ratio as a rate or comparison relation and in applying the concept of equivalent ratios in solving rate and comparison problems in working with per cent, in calculating area and volume, and in converting one unit of measurement into another. In grade 6, students are taught the ratio test.

($\frac{a}{b} = \frac{c}{d}$, if and only if ad = bc)

This enables them to work with many types of ratio problems.

References: S.T.A. Grade 6 page 141

S.T.A. Grade 6 Teacher's Guide page 144

Some examples of the application of ratio are as follows:

(a) 25% of n = 352

$$\frac{25}{100} = \frac{352}{n}$$

$$25\,n = 100 \times 352$$

$$n = \frac{\cancel{100}^{4} \times 352}{\cancel{25}}$$

(Note calculation is left until cancellation is possible.)

$$n = 1408$$

25% of 1408 = 352

(b) A piece of paper is in the shape of a rectangle 3 inches wide and 5 inches long. What is its area in square inches?

$$\frac{5}{1} = \frac{n}{3}$$

$$n = 15$$

$$\frac{5}{1} = \frac{15}{3} \text{ (check)}$$

The area of the paper rectangle is 15 square inches.

References: S.T.A. - Grade 6, page 154
S.T.A. - Grade 6, Teacher's Guide page 157

(c) 864 cu. in. = cu. ft.

$$\frac{1728}{1} = \frac{864}{n}$$

$$1728n = 864$$

$$n = \frac{864}{1728} = \frac{1}{2}$$

864 cu. inches = $\frac{1}{2}$ cu. ft.

(d) Mr. Castle bought 12 rosebushes for \$28.00. At this rate he paid how much for 3 rosebushes?

$$\frac{12}{28} = \frac{3}{X}$$

$$12X = 3 \times 28$$

$$x = \frac{3 \times 28}{12} = 7$$

or

$$\frac{12}{28} = \frac{3}{X}$$

$$\frac{12}{28} = \frac{3}{7} \text{ (Using equivalence)}$$

$$x = 7$$

He paid \$7.00 for 3 rosebushes

References: S.T.A. Grade 6, page 112
S.T.A. Grade 6, Teacher's Guide P. 120

The introduction and application of the concept of ratio, especially to per cent problems are a significant departure from the conventional mathematics program. Teachers in Grade seven should use the ratio concept wherever applicable. However, they may teach also the solution of percentage problems through the use of decimal fractions as enrichment. Teaching the solution of percentage problems using the ratio approach will require some modification of the present program. This modification and others are included in this guide.

GEOMETRY

Students entering grade 7 with a background of Gage Arithmetic will have been taught the concepts of perimeter and area of rectangles, squares, and parallelograms, and the surface area of a rectangular prism and of the volume of a rectangular prism. In addition, these students have been taught recognition of many types of plane figures and solids (e.g. cylinders, prisms, cones, spheres, and pyramids) In teaching the concept of area the Gage series used the concept of ratio as indicated in problem ,

DIVISION

The Gage series teaches the successive subtraction approach to division.

Examples:

$14472 \div 536$

```
536 | 14472 | 20
      10720 |
       3752 |  5
       2680 |
       1072 |  2
       1072 |
            | 27
```

$67.50 \div 1.8$

```
18 | 675.0 | 300
     5400  |  70
     1350  |
     1260  |
       90  |   5
       90  |
           | 37.5
```

However the traditional form of division is taught in Grade 6 as a possible development from the subtractive approach. This is treated as a side trip. In

any case it would not seem necessary to demand the subtractive approach if the student in grade 7 can divide the traditional way.

3. THE GRADE SEVEN MODIFIED MATHEMATICS PROGRAM

Since the Gage program in grade 6 goes beyond the traditional program in many respects, it follows that parts of the present text in grade 7, Winston Mathematics Book I will serve as oral review and some parts must be deleted entirely. It is expected that some added work which will serve as enrichment will become necessary. We will have to introduce new work from modern mathematics. Two possible modern concepts might include (1) number concepts, and (2) sets, sentences and variables as outlined in Preview of the Scott, Foresman Seventh Grade Mathematics Program of which you will have possession when they are made avaiable.

The grade seven course in mathematics will thus consist of work from the present text followed, if time permits, by a few modern topics.

Following is a guide to help the grade seven teacher in 1962 - 63.

Note that the use of ratio concept is expected whenever it can be applied.

Red-starred items in text usually indicates enrichment and should be used.

PAGE	TREATMENT
1	Regular, graph interpretation is new.
2-5,8,12,16,35-37	Review orally. If lesson indicates definite work in certain areas - then expand into this field of work.
6 & 7	Diagnostic Test and Follow - Up as usual. Note how the new approach has crept into the work.
9	Regular - Do enrichment on magic squares and possibly supply more challenging material of this nature.

PAGE	TREATMENT
28-29	Delete as page 20, except practice
30	Delete - This approach is not used, practice is satisfactory.
31 -32	Regular
33	Enrichment to be done - most will use or should use ratio
34	Regular - Number Quickies excellent
35	Review odd and even integers "Understanding Numbers" - NOTE an illustration will show that the statement could be true as on page 13.
38	Good diagnostic test
39	Chapter test OK

CHAPTER II - "How Well Do You Understand Fractions" Page 41 - 85

Page	Treatment
41 -48, 53,54 60,63,64,67,69-73 78	Review Orally - If weaknesses arise then more concentrated work is necessary.
49	Diagnostic Test OK
50	A general formula or equation should be developed for the situation. Such equations could be: $c = 20¢ + 5(n-\frac{1}{4})¢$ when n is quarter mile or better yet $c = 20 + 5(4n - 1)$ where n is the number of miles

Page		Treatment
77		Use ratio approach

Thus 5 (a):

$$2/3 \text{ of } ? = 46$$

$$\frac{2/3}{46} = \frac{1}{n}$$

$$2/3n = 46$$

$$n = 46 \times 3/2 = 69$$

79 - 83	OK	Use ratio where applicable

Special Notes:

P. 59. The rule for multiplying fractions by a whole number may be new to students from the Gage series. They should be used to this procedure:

$$2 \times \tfrac{3}{4} = \frac{2}{1} \times \tfrac{3}{4} = \frac{6}{4} = 1\,\frac{2}{4} = 1\tfrac{1}{2}$$

P. 68: Students from the Gage program may find the definition of ratio somewhat inadequate. They think of a ratio as expressing a rate or comparison.

P. 70 and onward: The rule for dividing fractions will be knew to Gage students. They have been taught to reduce the divisor to 1 by multiplying both.

Numeration and denominator by the reciprocal of the divisor. For example:

$\frac{3}{4} \div \frac{1}{3}$ Reciprocal of the divisor $\frac{1}{3}$ is $\frac{3}{1}$

$$\frac{\frac{3}{4} \times \frac{3}{1}}{\frac{1}{3} \times \frac{3}{1}} = \frac{\frac{9}{4}}{1} = 2\tfrac{1}{4}$$

Page	Treatment
140	See STA p. 243 - 246 delete this approach do exercises
141	Avoid use of decimal and common fractions together (eg) $.33\frac{1}{3}$
	Use ratio $\frac{1}{3} = \frac{X}{100}$, $X = \frac{100}{3} = 33.3$
	Delete books approach - do exercises
142 - 143	Regular but use ratio
144	Use ratio (eg) No. 3
	$\frac{3.64}{2.51} = \frac{x}{1}$, $x = 1.45 \frown 1.5$
145	Use ratio (eg) No. 1
	$\frac{1}{10.23} = \frac{4560}{x}$, x = ----------
146	Two methods both ratio
	$17500 + n = P$
	$\frac{17500}{100} = \frac{n}{18}$ Where n = increase
	$n = 3650$
	Population is now $P = 17500 + 3650 = 20650$
	<u>or</u>
	$\frac{17500}{100} = \frac{X}{118}$
	$X = 20650$
147	Probable Review
148	Regular Treatment
149	Delete Introduction - do exercises
150 - 151	Ratio approach
152	Use all methods but emphasize ratio
153 - 155	Rules deleted - Use first principles

Page	Treatment
156	Ratio as well
157	OK - using ratio
158	OK
159	Devise qurstions to test third type in grade 8 text.
160	Use first principle and logic
161 - 162	Review new approach
163 - 167	Excellent tests except for 3rd type which you should include.

NOTES:

Gage students have been taught that per cent involves a comparison - so many per hundred. The Ratio approach must be taught. The other approach in text might serve as enrichment.

P. 136 Gage students might argue that while 70% = $\frac{70}{100}$ it does not equal .70. They might insist that 70% = $\frac{.70}{1.}$

Having defined per cent as a ratio they may insist it cannot equal a number. This distinction applies throughout the chapter. For example on page 101 they would disagree that 1% means 1 hundreth (.01) and on page 137 they would disagree that per cent means hundreths or two decimal places. Rather they would say it means so many per hundred.

CHAPTER V:

Age Groups in Canada (Graphs) p. 169 - 193

Treat entire chapter in usual manner. Watch for possible uses of ratio.

NOTE: watch expressions in scales that say 1 disk = 10 years - rather, 1 disk represents 10 years. P. 174

The students from Gage have gone beyond the material in this chapter in some respects. eg area of a general parallelogram, volume of a cube. They have learned to calculate area using a ratio approach rather than following or substituting in a formula. For example, if asked to find the area of a piece of paper 4 1/3" X $2\frac{1}{2}$", the solution may look like this:

$$\frac{4\ 1/3}{1} = \frac{W}{2\frac{1}{2}}$$

$$W = 4\ 1/3 \times 2\tfrac{1}{2}$$

$$= \frac{13}{3} \times \frac{5}{2}$$

$$= \frac{65}{6}$$

$$= 10\ 5/6$$

$$\frac{4\ 1/3}{1} = \frac{10\ 5/6}{2\frac{1}{2}} \quad \text{check}$$

The area of the piece of paper equals 10 5/6 sq. in.

References: Seeing Through Arithmetic - Grade 6 page 186
" " " - Grade 6 Teacher's Guide, page 182

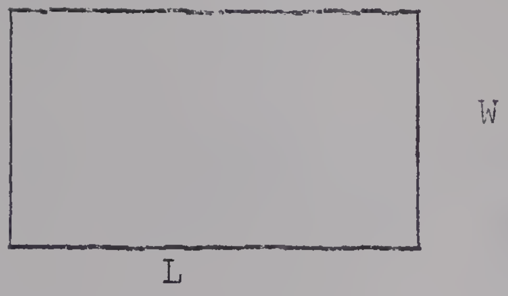
Page	Treatment
193 - 201	Regular
202 - 203	avoid use of 1" = 10' as in # 6
204 - 205	aviod rules use first principles. Rules may be a form of enrichment, however
206	# 4. Use ratio

$$\frac{5}{1} = \frac{n}{4}$$

$$n = 20$$

There are 20 sq. inches in the rectangle.

Page	Treatment
207	Delete top half of page up to and including # 1. Introduce formula A A = lw through ratio as enrichment.

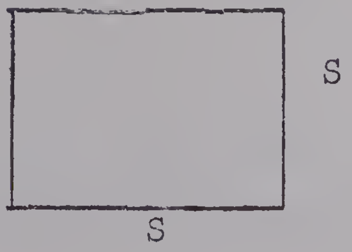


$$\frac{L}{1} = \frac{A}{W}$$

$$A = lw$$

Exercises 2 - 13 OK

208 - 209	OK. use ratio whenever applicable
210	Delete area of square Develop by ratio

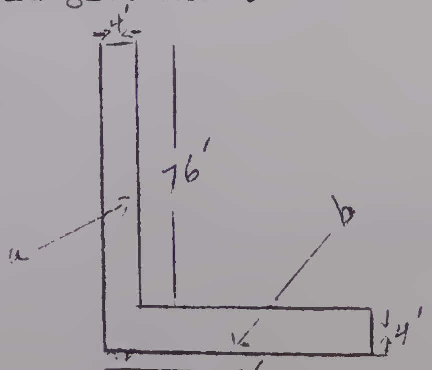


$$\frac{S}{1} = \frac{A}{S}$$

$$A = S^2$$

Magic square excellent enrichment- could give more.

211



$A = a + b$

$\frac{1}{76} = \frac{4}{a}$

$a = 76 \times 4$ (Do not workout)

$\frac{1}{51} = \frac{4}{b}$

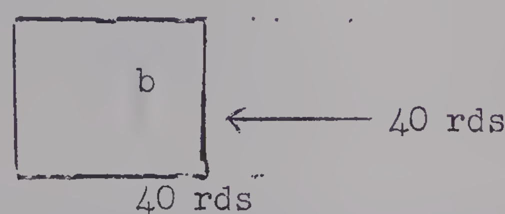
$b = 4 \times 51$

$A = (76 \times 4) + (4 \times 51)$

$A = 4(76 \quad 51) = \underline{\underline{508}}$

Area of Walk is 508 sq. ft.

212 # 7



Let a = number of sq. rd. in plot
Let b = number of acres in plot

$\frac{1}{160} = \frac{b}{a}$

Now $\frac{a}{40} = \frac{40}{1}$

$a = 40 \times 40$

Size of plot is 40 X 40 sq. rds

Now to find b

Continued on next page

Page	Treatment
251	Use ratio directly as follows #4/251

$$\frac{34}{100} = \frac{n}{360}$$

$$n = \frac{360 \times 34}{100} = 122.4$$

Army sector represents 122.4°

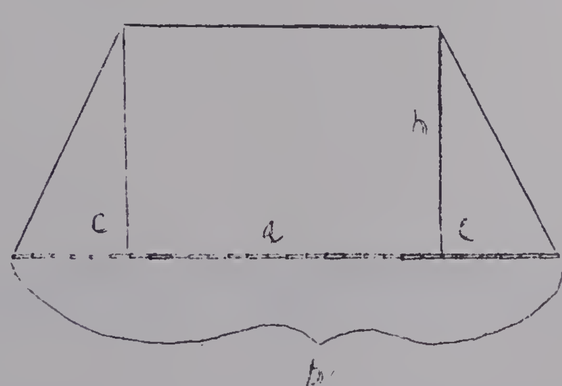
Air Force sector represents 122.4°

Remainder represents $(360^\circ - 244.8^\circ) = 115.2^\circ$

252 - 253	Regular
254 - 255	Use only limited number of formulae - circle
256 - 259	Regular

NOTES:

(1) Finding area of a trapezoid may be done simply from first principles. However at the grade level and ability the development of a formula could be taught as enrichment.



$$\text{Area} = ah + \tfrac{1}{2}hc + \tfrac{1}{2}hc$$

(Gestalt)

$$= ah + hc = h(a + c)$$

but $b = a + 2c$

$$b - a = 2c$$

$$\frac{b-a}{2} = c$$

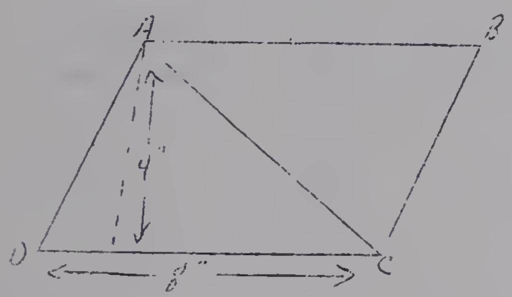
$$A = h\left(a + \frac{(b-a)}{2}\right)$$

$$= h\left(\frac{2a + b - a}{2}\right)$$

$$= \frac{h}{2}(a + b)$$

(ii) Formula for area of parallelogram should precede that of the triangle.

The development of both follows:



Area of llgm ABCD if found thus:

$$\frac{8}{1} = \frac{A}{4}$$

$$A = 32$$

Knowing this and after using cutouts proving in many cases to be true that a diagonal of a llgm forms two congruent or identical triangles one can proceed:

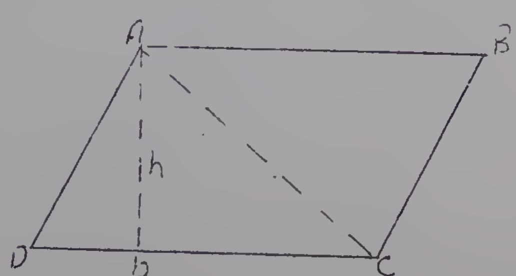
$$\frac{1}{2} = \frac{A}{32}$$

$$A = 16$$

Area of $\triangle$ is 16 sq. in.

NOTE: This is not a general proof because we have not proved that all diagonals form two identical triangle. This must be accepted or proved for enrichment.

Now proceeding to the more general (with the above assumption)



$$\frac{1}{b} = \frac{h}{A}$$

$$A = bh$$

Thence a formula for the area of $\triangle$ is

$$A = \frac{bh}{2}$$

(iii) # 6/248

Do not substitute numbers if this is to constitute a proof. You will only prove statement for one specific case. We need a GENERAL proof:

$C = 2\pi r$ - Note: 2π is a constant hence C will be affected the same as r.

Proof: $C = 2\pi r$ in the first instant
$C = 2\pi(2r)$ in the second instant
$= 4\pi r$

Note $4\pi r$ is twice $2\pi r$, hence circumfernce is doubled.

(iv) # 8/b/248

A general proof is as follows:

Let radius = "a" in the first case and C = circumference
Let radius = "b" in the second case and K = circumference

Then $C = 2\pi a$ and $k = 2\pi b$

Suppose $K \neq C$
then
$2\pi a \neq 2\pi b$

and ∴
$a \neq b$

but $a = b$ is given

$K \neq C$ is false

and $K = C$ follows.

<u>CHAPTER VIII</u> Family Budgets Page 261 - 299

The concept of volume of a rectangular solid is not new to the Gage students. The formula for the volume of a rectangular solid is not formally taught, however.

Page	Treatment
299 - 302	Regular - Ratio where applicable
303	# 5/303

$$\frac{450}{100} = \frac{n}{6} \quad \ldots\ldots\ldots\ldots$$

304	Same approach
305	Use two ratio situations

(eg) # 2/ 305

Interest for 1 year

$$\frac{5}{100} = \frac{n}{450}$$

$$n = \frac{\cancel{450} \times 5}{\cancel{100}} = 22.50$$

The rate is \$22.50 for 1 year

Thus for $1\frac{1}{2}$ years:

$$\frac{22.50}{1} = \frac{i}{1\frac{1}{2}}$$

$$i = \overset{11.25}{\cancel{22.50}} \times \frac{3}{\cancel{2}} = 33.75$$

Interest on \$450 at 5% for $1\frac{1}{2}$ years is \$33.75

306	Regular
307	Interest Formula - Delete - proof in text. Follow note at end of chapter guide:
308	As page 305

3/308

Interest for one year:

$$\frac{4}{100} = \frac{n}{500}$$

$$n = \frac{2000}{100} = 20$$

Interest for one year is \$20.00

Interest for 9 months

$$\frac{9}{12} = \frac{b}{20}$$

$$b = \frac{9 \times 20}{12} = 15$$

$\therefore$ Interest on \$500. at 4% for 9 months is \$15.00.

309 - 332

Ratio approach where applicable. Treat discounts and mark-ups as % increases & decreases. Commission is simple ratio relationship.

Note: Interest formula

I = interest for period desired
K = interest for one year
t = time of note expressed in years
p = principal
r = rate of interest on money for 1 year

(1) $\frac{K}{p} = \frac{r}{100}$ and (2) $\frac{t}{1} = \frac{I}{K}$

from (2) we obtain $I = tk$
" (1) " " $K = \frac{Pr}{100} = P \cdot \frac{r}{100}$

If r is to be expressed as a decimal, we obtain K = pr. Substituting K = pr in (2) we have:

I = tpr or I = prt

Students might use this formula only after they fully understand what it means.

GENERAL OBSERVATIONS

1. Rules: A number of rules appear in red print throughout the text. In most cases they should be deleted. The new mathematics is a mathematics of UNDERSTANDING rather than MEMORIZATION. Concepts or understandings for example relating to per cent increase, decrease, commissions etc must be fundamentally understood. If these are understood, problems of such nature can be worked from FIRST PRINCIPLES rather than from a rule or formula.

2. RATIO: This is one of the most potent tools in mathematics. To be fully understood, this concept must be applied whenever and wherever it is applicable. Teachers must therefore think ratio at all times and have students think ratio and use ratio at all times. It is thus urgently recommended that grade seven teachers using this program study thoroughly the concept of and approach to ratio in the grade 5 and 6 Gage STA series as well as the related pages in "Charting the Course for Arithmetic". Check thoroughly the approach using ratio to per cent, circle graphs, area and volume, interest etc.

3. Division: Students from the Gage series have been taught division through the subtractive division concept. However the traditional method of long division is presented in sixth grade (pages 162-164 and 235). When children meet it at this time, it takes on meaning that it otherwise could not have, for now they realize what the number fragments stand for. Students who would rather use the **traditional** method should not be discouraged and forced into using the new approach. The new approach is a method of teaching division with understanding. Once understanding of division is achieved the student would be able to use any algorism he chooses.

4. Decimal Multiplication and Division:

Teachers should check the pages of the STA grade 6 for the new Gage approach to decimal multiplication and division. Being a more sensible approach, it should not be pushed aside in favor of the traditional way.

5. Grade Six S.T.A. Content:

It would be well that teachers briefly review the content of the grade six S.T.A. This is found on page 286 of the grade six S.T.A.

6. "Mixed" Fractions:

The present grade 7 text, Winston Mathematics Bk. I combines decimals and fractions into a "mixed" numeral, eg, .331/3. Avoid this completely.

7. Percent:

Per cent is thoroughly and completely taught in grade 6 S.T.A.. Thus in grade seven these students will generally find the work on fundamentals of per cent REVIEW. However the approach to per cent is completely different as indicated in this guide. Per cent is taught on a ratio basis. If possible use this approach only and the methods indicated in Winston Mathematics Bk. I might serve as enrichment. Check sources for the philosoply of the approach. The "third case" formerly taught in grade 8 is taught in grade 6 S.T.A.. IT WILL BE NECESSARY TO SUPPLY EXERCISES FOR MAINTENANCE IN GRADE 7 FOR THIS "THIRD TYPE" AS IT IS NOT FOUND IN THE PRESENT TEXT.

8. Formulae:

In the past it was necessary in most classrooms to have students memorize formulae, seldom without understanding, and attempt to apply these to a problem

situation. Application to such problem situation was rather unfruitfull. In the "new" mathematics understanding is first. Therefore, if students understand the fundamentals of perimeter, area, volume, etc. they should need relatively few formulae (memorized). With the exception of the formulae for circumference and area of a cicle in grade 7, no other formulae should be required. Development of such other formulae would serve as enrichment only. Most problems of such nature can be solved from first principles. These are indicated in the guide. Ratio is used to solve most problems of area and volume in grade 7. This is entirely a new approach. Teachers must become familiar with this. See appropriate sources of information.

9. $A = \frac{1}{2}bh$:

The formula for the area of a triangle should be developed only after considerable work is done in the area of a parallelogram by ratio from basic principles. See development in guide.

10. Circle Graphs:

Now with the use of ratio students can compute directly from information given into number of degrees for the circle graph. DO NOT USE THE APPROACH IN "WINSTON MATHEMATICS BK. I".

11. Interest Formula:

The interest formula should be developed only after considerable practice as indicated in the appropriate section of the guide. Notice ratio appraach is used.

12. Sources of Information For Teachers:

1. This Guide - A Modified Program in Mathematics for Grade Seven - 1962-63 is most essential and should be kept in close contact during mathematics period.

2. Charting the Course for Arithmetic - Hartung and Van Engen very thoroughly gives the content of the S.T.A. series in grade 1 - 6 with appropriate philosoply, illustration, and comparisons. A most valuable booklet to digest.

3. Grade 5 and 6 - Seeing Through Arithmetic texts will show how the concepts are introduced and taught.

4. Preview of the Scott, Foresman Seventh - Grade Mathematics Program will provide an interesting preview of what can possibly be expected in junior high school mathematics in the future. Pages 41 - 43 may in part be used as enrichment ideas toward the end of the school year. Much planning must still be done along this line.

APPENDIX C

COUNTY OF BEAVER NO.9

A GUIDE FOR A MODIFIED PROGRAM IN GRADE EIGHT MATHEMATICS FOR USE WITH PRESENT WINSTON MATHEMATHICS BOOK I

WITH THE ADDITION OF SEVERAL ENRICHMENT UNITS

1963 - 64

Written and Compiled by
R.A. Gorrie
Supervisor of Instruction

COUNTY OF BEAVER # 9

A MODIFIED PROGRAM IN MATHEMATICS GRADE 8 1963-64

. OBJECTIVES AND PURPOSES:

(a) To provide for continuity of learning from the grade 6 STA series through the modified grade 7 program of 1962-63 to the completion of the grade 8 year.

(b) To provide and maintain experiences in the gestalt and ratio equation approach to problem solving.

(c) To provide experiences which will lend themselves to the more "modern" approach to teaching mathematics.

(d) To present experiences which will allow for individual differences and better understanding of mathematics.

(e) To provide for the objectives as outlined in the junior high school Handbook as well as the junior high school curriculum guide in mathematics.

. PROBLEMS:

The following is an abstract from the grade 7 guide of 1962-63.

A. PROBLEM SOLVING:

Students with a background from Study Arithmetic or the Ginn series of Arithmetic We Need have been taught to keep certain questions in mind when examining a problem. e.g. What am I asked to find? Should I add, subract, multiply or divide? Also in Ginn the "Cue Word" concept is taught in the methods of problem solving. However, our immediate problem concerns the new approach to problem solving as presented in Gage.

The Gage series spends much time presenting an "equation approach" to problem solving. It is suggested that students utilize a four-step procedure as follows:

1. Read the problem,analyze the situation and note the action that is taking place.
2. Express the situation in the form of a mathematical sentence or equation.
3. Compute to the side.
4. Answer the question asked in the problem.

Sometimes a check (substitution) is placed following the computation and before

the final statement of the answer is written.

For example, the solution to the following problem might be as indicated:

"Joan bought 2 packages of notebook paper at 37¢ each, and a package of pencils for 49¢. How much did she spend for all these things?

a + \$.49 = d	\$.37	\$.74
(2 x \$.37) + \$.49 = d	2	\$.39
\$.74 + \$.49 = d		
\$.74 + \$.49 = \$1.13	\$.74	\$1.13

Joan spent \$1.13 for all these things

References: S.T.A. - Grade 6, page 50
S.T.A. - Grade 6, Teacher's Guide page 67

Note: Students are encouraged to have the equation express the EXACT and ENTIRE situation in the problem and verbal statement other than the concluding one are discouraged. Few rules are given for finding the letter used in the equation.

. RATIO

This term has been scarcely referred to in Study Arithmetics. In the Ginn series the term is referred to and defined as a relation between two numbers. In the Gage series much time is spent developing the concept of ratio as a rate or comparison relation and in applying the concept of equivalent ratios in solving rate and comparison problems in working with per cent, in calculating area and volume, and in converting one unit of measurement into another. In grade 6, students are taught the ratio test.

$\left(\frac{a}{b} = \frac{c}{d}, \text{ if and only if } ad = bc \right)$

This enables them to work with many types of ratio problems.

References: S.T.A. Grade 6 page 141
S.T.A. Grade Teacher's Guide page 144

Some examples of the application of ratio are as follows:

( )

(a) 25% of n = 352

$\frac{25}{100} = \frac{352}{n}$

$25n = 100 \times 352$

$n = \frac{\cancel{100}^{4} \times 352}{\cancel{25}}$

(Note calculation is left until cancellation is possible.)

$n = 1408$

25% of 1408 = 352

(b) A piece of paper is in the shape of a rectangle 3 inches wide and 5 inches long. What is its area in square inches?

$\frac{5}{1} = \frac{n}{3}$

$n = 15$

$\frac{5}{1} = \frac{15}{3}$ (check)

The area of the paper rectangle is 15 square inches.

References: S.T.A. - Grade 6, page 154
S.T.A. - Grade 6, Teacher's Guide page 157

(c) 864 cu. in. = cu. ft.

$\frac{1728}{1} = \frac{864}{n}$

$1728n = 864$

$n = \frac{864}{1728} = \frac{1}{2}$

864 cu. inches = $\frac{1}{2}$ cu. ft.

(d) Mr. Castle bought 12 rosebushes for $28.00. At this rate he paid how much for 3 rosebushes?

$\frac{12}{28} = \frac{3}{x}$

$12x = 3 \times 28$

$x = \frac{3 \times 28}{12}$

or

$\frac{12}{28} = \frac{3}{x}$

$\frac{12}{28} = \frac{3}{7}$ (Using equivalence)

$x = 7$

He paid $7.00 for 3 rosebushes.

References: S.T.A. Grade 6, page 112
S.T.A. Grade 6, Teacher's Guide P. 120

The introduction and application of the concept of ratio, especially to per cent problems are a significant departure from the conventional mathematics program. Teachers in Grade seven should use the ratio concept wherever applicable. However, they may teach also the solution of percentage problems through the use of decimal fractions as enrichment. Teaching the solution of percentage problems using the ratio approach will require some modification of the present program. This modification and others are included in this guide.

C. GEOMETRY

Students entering grade 7 with a background of Gage Arithmetic will have been taught the concepts of perimeter and area of rectangles, squares, and parallelograms, and the surface area of a rectangular prism and of the volume of a rectangular prism. In addition, these students have been taught recognition of many types of plane figures and solids (e.g. cylinders, prisms, cones, spheres, and pyramids). In teaching the concpt of area the Gage series used the concept of ratio as indicated in problem.

D. DIVISION

The Gage series teaches the successive subtraction approach to division.

Examples:

```
      14472 ÷ 536                          67.50 ÷ 1.8

     536 ‾‾‾‾‾‾‾‾‾‾                     18 ‾‾‾‾‾‾‾‾‾‾
         14472       20                     675.0      300
         10720                              5400        70
          3752        5                     1350
          2680                              1260
          1072        2                       90         5
          1072       __                       90      _____
                     27                               37.5
```

However the traditional form of division is taught in Grade 6 as a possible development from the subtractive approach. This is treated as a side trip. In any case it would not seem necessary to demand the subtractive approach if the student in grade 7 can divide the traditional way.

F. REPETITION:

Most of the problem solving can be done by means of ratio. Therefore the prime purpose of the problem exercises is to apply the concept learned to problem situations. It is not the purpose to do computation as busy work. Since ratio is not new in any situation much repetition will come about unless teachers are careful in the selection of problems. Maintain the application of the concept. DO NOT OVERDO exercises.

G. PRE-TEST:

Since pages 1 - 168 are actually a review of the grade 7 text now that ratio work is applied, a pre-test has been prepared to cover this area. The pre-test will indicate weaknesses in various areas, and teacher emphasis on specific materials will then be clearly defined.

H. NEW MATERIAL:

The section on geometry beginning on page 229 has an introductory section dealing with the concepts of point and line and other sequential concepts. A seminar will be held regarding this material and other "modern" topics later in the year. The references listed in the guide should be looked at regarding this concept.

I. NEW TOPICS:

Since a great deal of the material in chapters I and II is to be deleted a new unit is to be introduced before proceeding according to the following guide. The unit is entitled "Numeration Systems." Following the work in this unit teachers will proceed with the order in the guide making applications and references when possible to numeration system.

Other new topics which will be introduced later in the year after page 321 include:

(1) System of Natural Numbers and the Number line

(2) Properties of the Natural Numbers

(3) Extension to the Rational Numbers and Number line

(4) Properties of the Rational Numbers

(5) The irrationals

(6) Extension to the Real Numbers and Number line

(7) Properties of the Real Numbers

(8) Mathematical Systems

Depending on the time factor some or all of these units will be incluced in the grade 8 year along with the modification of the Winston Text. These units will be prepared cooperatively with Junior High school teachers in order to prepare teachers for this work.

J. EQUATIONS AND SQUARE ROOTS:

This material beginning on page 322 is to be deleted and later introduced after completion of the unit on rational numbers and irrational numbers.

K. BACKGROUND:

Teachers of grade 8 should review the grade 7 guide of 1962-63 to become extremely familiar with the processes and concepts developed from the "new" viewpoint which will be maintained and extended in the grade 8 year.

L. COMMUNITY RESOURCES:

In order to keep abreast of times and have material and facts current it becomes necessary to call on community resources from many of the topics as suggested in the guide. Local insurance agents, post masters, village, town and county secretaries are among such resources.

B. THE GRADE EIGHT MODIFIED MATHEMATICS PROGRAM

Introduction: Enrichment unit on Numeration Systems precedes Chapter I. After its completion apply concepts where applicable to the following pages especially of Chapters I and II.

CHAPTER I- "How Well do You Understand Whole Numbers and Common Fractions"

PAGE	TREATMENT
1	Abacus - most important to understanding grouping by 10's - do detail study.
2	Short review - few exercises
3	Oral review - basic understandings
4	Short review - few exercises
5	Measurement - most important concept
6 - 9	Delete
10	Calculating machines - Stress principle of operation
11 - 14	Delete except number quickies page 14
15 - 17	Delete except - casting out 9's operation
18 - 19	Delete
20	Review orally with some idea of derivation or use of chosen words.
21	Review but with new emphasis

1. $\frac{12}{16} = \frac{12 \div 4}{16 \div 4} = \frac{3}{4}$ <u>Stress</u> division of both numerator and denominator by the same number.

2. $\frac{8}{3} = \frac{3}{3} + \frac{3}{3} + \frac{2}{3} = 1 + 1 + \frac{2}{3} = 2\frac{2}{3}$ Stress the additive aspect or sub-group aspect of a number i.e. 8 is 3+3+2, or 6+2, etc.

   or $\frac{8}{3} = 2\frac{2}{3}$ because $\frac{8}{3}$ represents division of 8 by 3 <u>STRESS</u>

3. $\frac{5}{8} = \frac{x}{24}$ $\qquad$ $\frac{5}{8} = \frac{?}{24}$

   $8x = 5 \times 24$ or 24 is 3×8

   $x = \frac{5 \times \cancel{24}^{3}}{\cancel{8}} = 15$ $\qquad$ $\therefore\ ? = 3 \times 5 = 15$

   $\frac{5}{8} = \frac{15}{24}$ equivalence relation

Do not memorize any rule - use first principles.

22 - 26	Delete

PAGE	TREATMENT
27	Rules stating changing of measures must be ignored completely Use first principles and ratio. (ex.) 8 min. = ? sec. $\frac{1}{60} = \frac{8}{x}$ $\frac{\text{1 minute}}{\text{60 sec.}}$ $\frac{\text{8 minutes}}{\text{x sec.}}$ Note pattern of ratio $x = 8 \times 60 = 480$ 8 min. = 480 sec.
28	Review
29 - 30	Delete
31	Review vocabulary - number quickies for enrichment
32	Majority vote Study but emphasize definition as stated. Note this often misconstrued concept of majority of 1000 votes cast the distribution is A-400, B-300, C-200. A obtained the greatest amount but not a majority by definition STRESS
33 - 35	Delete
36	Review and stress concept
37	Delete questions 1 - 4 do 5 - 22 or parts
38	Review
39	Delete
40	Good oral review
41	Use only if necessary

Notes: CHAPTER I

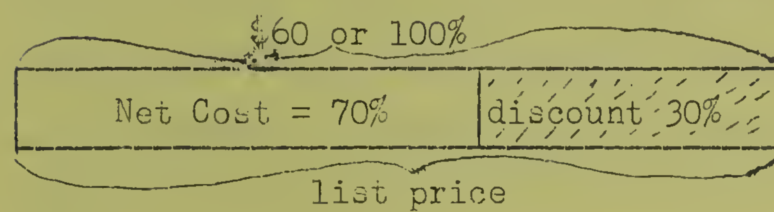
Much of this chapter is review. Pre-test will reveal any necessary study in a particular area. 8 - 10 lessons should normally be sufficient. Several concepts need definite emphasis. Stress these particularly. Number quickies are excellent enrichment if done orally. Use ratio approach whenever per cent is indicated.

PAGE TREATMENT

(e.g.) #5

List price \$60, % discount 30% Net cost?

If the list price is 100% and discount is 30% - net cost is 70% (remaining part)



_ Show with chart

Net cost + discount = list price

$\frac{70}{100} = \frac{n}{60}$ etc.

120 Successive discounts - review if necessary especially the principle

121 Vocabulary O.K. rest excellent orally

122 Delete

123 Chapter Test O.K. if necessary

124 Delete

125 Test Problem Solving O.K. if necessary

Notes:

Much of this chapter is maintenance of ratio methods of problem solving. Use your discretion in all cases.

CHAPTER IV - "More About Formulas"

127 Review of grade 7 only if necessary. Note "A" not "The" formula for perimeter of a square is P = 4s where P is the number of units of perimeter and s is the number of units of length of the side of a square. (All formulae should require qualifications.) Another formula for perimeter of a square might be T = 4b where T represents the total number of units of length of all sides and b represents the number of units of length in each boundary line, etc.............

PAGE | TREATMENT

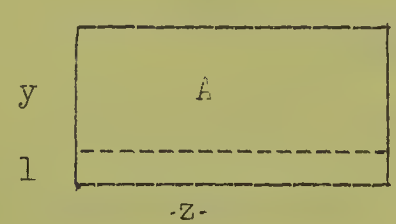
Formulae should not be memorized by rote. Only the basic principle or understanding should be learned and then put into a formula as the student may choose. This is a general principle to be used in all cases of mensuration. Specifically "perimeter" must be understood as the total number of units in a boundary. A formula may be developed for any or every case.

128 The most significant point on this page would be reverse of the distributive law over addition (factoring) rather than the point of perimeter. In grade 6 and some of 7, students have learned to remove brackets. Now they should learn the fundamentals of factoring. Further examples might be introduced.

129 After a review of the principle of area using ratio students may then simply use a formula $A = lw$. A formal proof of the formula should be required from time to time.

Begin by using the general then to the specific.

e.g.



Let z represent # of units of length and y represent # of units of width. A represents the # units of area.

$$\therefore \frac{z}{1} = \frac{A}{y}$$

$$A = yz$$

$$\text{where } y = z$$

$$A = y \cdot y = y^2$$

130 - 131 Maintenance only if necessary

132 Delete

133 Maintenance only - use ratio

PAGE	TREATMENT

Notes:

It would be advisable to study carefully the approach used in attacking most of these problems in grade 7. See guide grade 7. THIS IS MOST IMPORTANT BECAUSE OF THE REVISION OF SEQUENCE AND EMPHASIS.

CHAPTER V - "Interest"

165 - 167 The concept of simple interest and the formula I = prt discussed in grade 7 - see grade 7 guide. Treatment as review only *if* necessary. Delete rule and method on page 167. Develop I = prt for understanding. Allow use of formula only after understanding.

168 Usual treatment except application of ratio concept. Use local bank for resource materials.

e.g. # 3^d page 168

$$\frac{\frac{3}{4}}{100} = \frac{n}{760} \qquad \text{What is the unit of } n?$$

$$100\,n = 760 \times \frac{3}{4}$$

$$n = \overset{19}{\cancel{760}} \times \frac{3}{\cancel{4}} \times \frac{1}{\cancel{100}} = \frac{57}{10} = 5.7$$

The fee will be $5.70.

169 Delete rule - use first principles from ratio idea.

#5/169

$$\frac{15}{750} = \frac{x}{100} \qquad \text{Where } x \text{ is interest for } \tfrac{1}{2} \text{ year}$$

$$x = 2$$

Then the interest for $\frac{1}{2}$ year is 2%

Thus it follows since interest is better stated on a per annum bases that:

PAGE | TREATMENT

$$\frac{\frac{1}{2}}{1} = \frac{2}{i}$$

$$\tfrac{1}{2}i = 2$$

$$i = 2 \times \frac{2}{1} = 4$$

Interest for 1 year is 4%.

or more advanced the following procedure:

$$\frac{15}{y} = \frac{6}{12}$$ $15 is the interest for 6 months
y is the interest for 12 months.

$$y = 30$$

Then proceed to:

$$\frac{30}{750} = \frac{i}{100}$$ If students can readily see through ratio that interest will be $30 for twice the time, miss step 1 and proceed to the second.

170 - 188 Ratio application and delete practice P. 175, 177, 185, 188 unless it appears necessary and then only selected exercises. Use local resources such as bank, installement buying forms and brochures, N.H.A., local secretary - treasurer's office

Notes:

Students should be able to do the computation involved using the rate approach. However there are important concepts about finances that should be the ultimate goal of this chapter.

CHAPTER VI - "Compound Interest and Savings"

191 Oral work

192 Set up problems 5 & 6 and others similar to it is a pass-book style.

	#5/192 Date	Part.	Cr.	Dr.	Bal.
Interest period	July 1/62	D	1486		1486
is ½ year.	Aug. 15/62	W		250	1236
Lowest balance	Sept. 30/62	D	100		1336
during period	Nov. 1/62	D	200		1536
is $1236.	Jan. 1/63	Int.	?		?
Calculate on present bank rates!					

PAGE	TREATMENT
193	Use gestalt approach to problems.

Questions? What is the rate? We want the fee! What is the total amount of notes? The rate is 3/4%, the fee is "f" and the total amount is "t". Then:

$\frac{3/4}{100} = \frac{f}{t}$ What units are f and t? Does it make any difference?

and

$t = 6(10) + 9(20) + 130(50) + 42(100).$

A proper way of writing these compound problems would be:

$$\frac{3/4}{100} = \frac{f}{t} \wedge t = 6(10) + 9(20) + 130(50) + 42(100).$$

Which must I solve first? Why?

194 This principle is becoming out-moded. Treat from first principles and contrast and compare with simple interest.

Using the gestalt approach (e.g.) #8/195:

$d = t-s$ where d is the difference, t and s are amounts of compound and simple interest respectively:

$$\frac{4}{100} = \frac{x}{100} \wedge \frac{4}{100} = \frac{y}{100 + x} \wedge \frac{4}{100} = \frac{s}{100 + x+y} \wedge$$

$t = x + y + s$ where x, y and s is interest for first, second, & third year respectively, and t is the total interest over the 3 years.

$i = prt$

$i = 100 \times \frac{4}{100} \times 3 = 12$

Simple interest amounts to $12.

Difference will be $d = t - 12$..............

196 - 197 O.K. use ratio application

198 Use first principles ONLY. Only formula necessary is those of the circle and the area of a triangle.

199 Delete practise unless required. Number Quickies O.K.

e.g. #14/ 294

$$\frac{80}{100} = \frac{x}{12500} \quad \wedge \quad \frac{4.18}{100} = \frac{y}{x}$$

where x is assessed valuation and y is the tax

295 Delete rule regarding changing of mills to percent and vice - versa. Use ratio idea:

a mill is 1/10 cent. Thus 10 mills/cent and 1000 mills/dollar or 1000 mills/100 cents.

Then 2.58% converted to mills will be:

$$\frac{2.58}{100} = \frac{x}{1000}$$ and oonversely

25.8 mills converted to % is

$$\frac{25.8}{1000} = \frac{x}{100}$$

296 - 297 O.K. Use recent figures and applied calculations with ratio.

298 Excellent review.

Note #8. Correct Examples do not prove these statements but by extending the pattern of thought one can only assume that this appears possible. A <u>General Proof</u> is required. #8a could be proved thus.

Let a, b, c be numbers.

Then: $\frac{a}{b}$ is a fraction and multiplying divisor and dividend by the same number c we have: $\frac{a \times c}{b \times c}$

Assume the statement to be false. Then $\frac{a \times c}{b \times c} \neq \frac{a}{b}$ or $\frac{ac}{bc} \neq \frac{a}{b}$

Apply ratio test: $acb \neq abc$

$$\frac{acb}{ab} \neq \frac{abc}{ab}$$

$c \neq c$ which is contrary to what is

PAGE	TREATMENT
	given. $\therefore$ statement is true.
	or #(f)/298
	Let the odd number be $2a + 1$
	Then the even number must be $2a$
	$2a\ (2a + 1) = 4a^2 + 2a$. Both have factor 2, $\therefore$ product is even. Statement is false. Note however that by substitution and trying will <u>not prove</u> the statement for <u>every</u> case.
299	Delete practice unless necessary. Number quickies O.K.
300 - 301	Income Tax. Apply ratio exclusively. Obtain most recent forms of both farm and business types as well as personal from the local post office. Table on Taxable Income is out-dated.
302	Amusement Taxes - Obtain recent information from provincial government.
303 - 305	O.K. Ratio application
306	Delete all rules concerning calculation of bonds - use ratio application.
	Thus #1/306: $\frac{102\frac{1}{2}}{100} = \frac{x}{1000}$

307	Delete
308	O.K. Apply ratio
309	O.K. Apply ratio
310	Delete
311	O.K.

CHAPTER X - "Indirect Measurement and Equations"

313 - 315	Practice if needed - Grade 6 topic for these students - Oral work only.

GENERAL OBSERVATIONS

These following general observations are taken from the grade 7 guide for your convenience. Some additions for grade 8 are made.

1. Rules: A number of rules appear in red print throughout the text. In most cases they should be deleted. The new mathematics is a mathematics of UNDERSTANDING rather than MEMORIZATION. Concepts or understandings for example relating to per cent increase, decrease, commissions, etc. must be fundamentally understood. If these are understood, problems of such nature can be worked from FIRST PRINCIPLES rather than from a rule or formula.

2. RATIO: This is one of the most potent tools in mathematics. To be fully understood, this concept must be applied whenever and wherever it is applicable. Teachers must therefore think ratio at all times and have students think ratio and use ratio at all times. It is thus urgently recommended that grade seven teachers using this program study thoroughly the concept of and approach to ratio in the grade 5 and 6 Gage STA series as well as the related pages in "Charting the Course for Arithmetic". Check thoroughly the approach using ratio to per cent, circle graphs, area and volume, interest, etc.

3. Division: Students from the Gage series have been taught division through the subtractive division concept. However the traditional method of long division is presented in sixth grade (pages 162 - 164 and 235). When children meet it at this time, it takes on meaning that it otherwise could not have, for now they realize what the number fragments stand for. Students who would rather use the traditional method should not be discouraged and forced into using the new approach. The new approach is a method of teaching division with understanding. Once understanding of division is achieved the student would be able to use any algorism he chooses.

DECIMAL MULTIPLICATION AND DIVISION: Teachers should check the pages of the STA grade 6 for the new Gage approach to decimal multiplication and division. Being a more sensible approach, it should not be pushed aside in favor of the traditional way.

GRADE SIX S.T.A. CONTENT: It would be well that teachers briefly review the content of the grade six S.T.A. This is found on page 286 of the grade six S.T.A.

6. "MIXED"FRACTIONS: The present grade 7 text, Winston Mathematics Bk. I combines decimals and fractions into a "mixed" numeral, e.g., .331/3. Avoid this completely.

7. PER CENT: Per cent is thoroughly and completely taught in grade 6 S.T.A. Thus in grade seven these students will generally find the work on fundamentals of per cent REVIEW. However the approach to per cent is completley different as indicated in the guide. Per cent is taught on a ratio basis. If possible use this approach only and the methods indicated in Winston Mathematics Bk. I might serve as enrichment. Check sources for the philosophy of the approach. The "third case" formerly taught in grade 8 is taught in grade 6 S.T.A. IT WILL BE NECESSARY TO SUPPLY EXERCISES FOR MAINTENANCE IN GRADE 7 FOR THIS "THIRD TYPE" AS IT IS NOT FOUND IN THE PRESENT TEXT.

FORMULAE: In the past it was necessary in most classrooms to have students memorize formulae, seldom without understanding, and attempt to apply these to a problem situation. Application to such problem situation was rather unfruitfull. In the "new" mathematics understanding is first. Therefore, if students understand the fundamentals of perimeter, area, volume, etc. they should need relatively few formulae (memorized). With the exception of the formulae for circumference and area of a circle in grade 7, no other formuale should be required. Development of such other formulae would serve as enrichment only. Most problems of such nature can be solved from fi- principles. These are indicated in the guide. Ratio is used to solve most problems of area and volume in grade 7. This is entirely a new approach. Teachers must become familiar with this. See appropriate sources of information.

9. $A = \frac{1}{2}bh$: The formula for the area of a triangle should be developed only after considerable work is done in the area of a parallelogram by ratio from basic principles. See development in guide.

10. CIRCLE GRAPHS:

Now with the use of ratio students can compute directly from information given into number of degrees for the circle graph. DO NOT USE THE APPROACH IN "WINSTON MATHEMATICS BK. I".

11. INTEREST FORMULA:

The interest formula should be developed only after considerable practice as indicated in the appropriate section of the guide. Notice ratio approach is used.

12. SOURCES OF INFORMATIONFOR TEACHERS:

1. This Guide - A Modified Program in Mathematics for Grade Eight - 1963-64 is most essential and should be kept in close contact during mathematics period.
2. Charting the Course for Arithmetic - Hartung and Van Engen very thoroughly gives the content of the S.T.A. series in grade 1 - 6 with appropriate philosophy, illustration, and comparisons. A most valuable booklet to digest.
3. Grade 5 and 6 - Seeing Through Arithmetic texts will show how the concepts are introduced and taught.
4. Preview of the Scott, Foresman Eighth - Grade Mathematics Program will provide an interesting preview of what can possible be expected in junior high school mathematics in the future. Pages 41 - 43 may in part be used as enrichment ideas toward the end of the school year. Much planning must still be done along this line.

Proof: Repeated from the grade 7 guide is the concept of proof. Many exercises in the grade 7 text and grade 8 text ask for examples to show that a statement is true or false. Be reminded that by substituting numbers in examples does not make a statement generally true if the substitution does. However one counter-example is all that is necessary to prove validity or invalidity. Impress upon students that correct

substitution proves the statements for that case only in which substitution is made. An infinite number of correct substitutions does not guarantee the next substitution to be correct. Only a general proof (algebraic) will suffice or a counter example.

Compound Problems: Students at the grade 8 level should be introduced to setting up compound conditions from problems. An example from the guide: #14/294

$$\frac{80}{100} = \frac{x}{12500} \quad \wedge \quad \frac{4.18}{100} = \frac{y}{x}$$

This "system" of equations provides all the necessary information from the situation. Whenever two steps are required in ratio type problems use this approach. The symbol "$\wedge$" means "and". Sometimes the symbol "$\vee$" meaning "or" will become necessary.

BIBLIOGRAPHY

1. A Modified Program in Mathematics for Grade Seven - 1962-63
   County of Beaver #9.

2. Charting the Course for Arithmetic - Hartung and Van Engen

3. Seeing Through Arithmetic Grade 5 and Grade 6 - Hartung and Van Engen

4. 24th Yearbook NCTM The Growth of Mathematical Ideas 1959

5. Basic Concepts of Elementary Mathematics, Schauf - Wiley & Sons.

6. Seeing Through Mathematics - Hartung & Van Engen, W.J. Gage & Co.

7. Others to be added later in the year.

APPENDIX D

STATISTICAL FORMULAE

$$s^2 = \frac{N\sum X^2 - (\sum X)^2}{N^2}$$

where: s -- is the pooled unbiased varaince estimate

N -- is the size of the sample

X -- is the observation score

$$t = \frac{\overline{X}_1 - \overline{X}_2}{\sqrt{s^2/n_1 + s^2/n_2}}$$

$$df = n_1 + n_2 - 2$$

where: t -- is the t-ratio

X_1 -- is the mean of the first sample

X_2 -- is the mean of the second sample

s^2 -- is the pooled unbiased variance estimate

n_1 -- is the size of the first sample

n_2 -- is the size of the second sample

RAW SCORES FOR ALL STUDENTS

The scores and other data for each student are listed on the following pages. Column numbers are coded thus:

Column 1 -- Identification Number -(TT-100-400),(MT-500-800)

Column 2 -- OTIS I.Q.

Column 3 -- PS8 Total Score

Column 4 -- PS8 Rate Score

Column 5 -- PS8 Non-Rate Score

Column 6 -- PS8 Single-Step Score

Column 7 -- PS8 Multiple-Step Score

Column 8 -- M9 Stanine Score

Column 9 -- SCAT Total Score

Column 10-- SCAT Non-Verbal Score

Column 11-- SCAT Verbal Score

Column 12-- Age in Months

Column 13-- Sex (Male-1), (Female-0)

Column 14-- Time in Minutes to Complete PS8 Test

Column 15-- Ability Group Code 1,2,3,4,5 for HI,HA,AV,LA,LO

Column 16-- Time Group Code 2,3,4, for FW,AW,SW

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
101	099	17	12	5	4	13	7	081	42	39	169	1	098	3	3
102	120	18	12	6	5	13	7	082	54	38	163	0	092	1	3
103	110	15	11	4	5	10	8	079	44	35	168	1	105	2	4
104	115	14	11	3	6	08	6	087	48	39	162	0	100	2	3
105	111	11	08	3	5	06	3	069	44	25	178	1	103	2	4
106	096	18	14	4	5	13	7	063	25	38	173	1	118	3	4
107	096	11	07	4	3	08	1	057	30	27	174	1	088	3	3
108	116	17	11	6	3	14	6	088	48	40	165	0	096	1	3
109	115	18	13	5	6	12	6	082	39	43	182	1	116	2	4
110	124	19	14	5	5	14	8	090	46	44	168	1	082	1	3
111	116	08	05	3	4	04	3	067	46	21	163	1	089	1	3
112	111	15	11	4	4	11	8	090	48	42	168	0	118	2	4
113	113	09	08	1	3	06	4	048	28	20	166	1	091	2	3
114	116	17	13	4	5	12	7	089	48	41	165	1	101	1	3
115	119	14	12	2	5	09	8	093	47	46	166	0	089	1	3
116	098	19	13	6	5	14	5	052	26	26	181	1	121	3	4
117	113	08	05	3	5	03	5	055	26	29	173	1	085	2	3
118	103	19	14	5	4	15	7	077	42	35	171	1	091	3	3
119	103	11	09	2	5	06	5	064	32	32	172	0	112	3	4
120	120	16	11	5	2	14	8	097	51	46	162	1	094	1	3
121	118	13	09	4	5	08	5	082	52	30	161	1	073	1	2
122	115	18	15	3	5	13	9	092	50	42	168	0	093	2	3
123	098	17	12	5	6	11	7	065	33	32	175	1	091	3	3
124	103	15	10	5	3	12	8	071	27	44	174	0	087	3	3
125	112	19	12	7	5	14	8	088	41	47	167	1	062	2	2
126	074	11	07	4	5	06	2	044	21	23	201	1	088	5	3
127	114	18	13	5	6	12	7	089	42	47	171	1	114	2	4
128	124	24	19	5	6	18	9	104	55	49	164	1	102	1	3
129	115	18	13	5	4	14	5	083	47	36	169	1	112	2	4
130	110	18	13	5	6	12	8	075	35	40	170	1	093	2	3
131	107	14	08	6	3	11	7	059	21	38	163	1	105	2	4
132	101	17	12	5	5	12	5	078	37	41	178	1	074	3	2
133	093	15	11	4	4	11	4	060	25	35	170	1	112	4	4
134	121	22	16	6	5	17	9	103	54	49	169	1	083	1	3
135	124	21	16	5	6	15	8	099	55	44	169	0	097	1	3
136	095	08	03	5	2	06	4	058	32	26	191	1	084	4	3
137	111	14	11	3	5	09	6	084	43	41	168	0	091	2	3
138	106	18	16	2	6	12	8	091	49	42	168	0	098	2	3
139	100	13	10	3	5	08	7	067	33	34	163	1	094	3	3
140	103	15	10	5	4	11	6	068	32	36	162	0	097	3	3
141	108	18	12	6	5	13	6	072	32	40	168	1	097	2	3
142	098	08	07	1	3	05	2	060	34	26	171	1	104	3	4
143	077	04	01	3	1	03	1	056	39	17	187	1	089	5	3
144	084	11	08	3	4	07	4	046	27	19	186	1	125	5	4
145	098	14	10	4	6	08	6	053	20	33	166	1	101	3	3

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
146	112	18	15	3	5	13	5	085	48	37	167	1	110	2	4
147	113	20	16	4	5	15	6	090	46	44	163	0	092	2	3
148	111	19	15	4	6	13	7	089	47	42	171	1	103	2	4
201	092	11	09	2	4	07	5	059	24	35	169	0	108	4	4
202	111	17	13	4	5	12	5	094	52	42	166	1	107	2	4
203	100	16	11	5	5	11	5	053	23	30	172	0	092	3	3
204	064	00	00	0	0	00	1	031	18	13	203	1	108	5	4
205	095	12	08	4	2	10	3	066	31	35	170	1	085	4	3
206	113	14	10	4	5	09	5	079	51	28	168	1	089	2	3
207	100	16	13	3	5	11	5	085	48	37	163	1	091	3	3
208	103	12	09	3	2	10	6	070	39	31	176	0	084	3	3
209	110	20	15	5	6	14	8	089	48	41	170	0	088	2	3
210	098	12	09	3	6	06	5	047	25	22	168	0	085	3	3
211	091	10	08	2	3	07	4	047	15	32	186	0	093	4	3
212	105	20	15	5	6	14	7	073	31	42	168	0	074	3	2
213	109	15	11	4	6	09	7	079	46	33	173	0	087	2	3
214	123	19	15	4	3	16	5	097	52	45	162	0	079	1	2
215	082	06	04	2	3	03	2	047	19	28	169	1	092	5	3
216	103	07	06	1	3	04	5	059	24	35	164	1	085	3	3
217	097	09	07	2	4	05	2	059	26	31	179	0	087	3	3
218	109	12	10	2	5	07	4	078	41	37	166	0	085	2	3
219	095	13	11	2	3	10	4	062	27	35	179	0	092	4	3
220	095	07	06	1	5	02	3	044	21	23	168	0	092	4	3
221	112	20	17	3	6	14	6	092	44	48	166	1	092	2	3
222	109	18	14	4	5	13	6	093	48	45	166	0	095	2	3
223	079	05	02	3	1	04	1	035	20	15	176	1	086	5	3
224	108	10	07	3	4	06	4	069	44	25	162	0	084	2	3
225	097	18	15	3	5	13	6	075	38	37	173	1	084	3	3
226	102	12	08	4	4	08	3	060	28	32	170	0	076	3	2
227	109	13	11	2	4	09	5	072	36	36	162	0	092	2	3
228	067	02	02	1	1	01	1	018	06	12	193	0	085	5	3
229	104	13	09	4	6	07	4	063	34	29	169	0	098	3	3
230	093	13	10	3	4	09	5	062	22	40	165	0	085	4	3
231	119	16	13	3	4	12	6	089	45	44	163	1	073	1	2
232	106	10	07	3	3	07	4	062	36	26	166	0	085	2	3
233	094	05	04	1	4	01	4	052	27	25	158	0	083	4	3
234	097	08	05	3	5	03	4	060	26	34	175	0	084	3	3
235	105	11	08	3	5	06	5	053	19	34	170	0	074	3	2
301	111	17	14	3	5	12	8	083	41	42	170	0	085	2	3
302	083	05	05	0	3	02	1	041	23	18	177	1	097	5	3
303	104	14	11	3	5	09	4	074	51	23	185	1	096	2	3
304	098	16	10	6	5	11	5	069	33	36	169	0	097	3	3
305	107	22	17	5	6	16	7	082	41	41	165	1	087	2	3
306	110	12	10	2	4	08	5	080	52	28	171	0	081	2	3

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
307	106	17	13	4	6	11	6	075	32	43	173	1	081	2	3
308	098	12	10	2	4	08	5	071	41	30	172	0	101	3	3
309	092	09	04	5	3	06	5	070	39	31	172	0	115	4	4
310	086	03	02	1	2	01	1	040	22	18	186	1	068	4	2
311	084	15	12	3	4	11	6	051	19	32	169	1	093	5	3
312	096	12	10	2	5	07	6	047	18	29	169	0	086	3	3
313	103	14	11	3	6	08	6	068	38	30	173	1	083	3	3
314	101	14	12	2	6	08	5	062	34	28	173	0	073	3	2
315	116	20	15	5	6	14	9	098	50	48	171	0	093	1	3
316	096	02	02	0	1	01	1	057	48	09	165	0	078	3	2
317	108	16	11	5	2	14	6	079	44	35	163	1	064	2	2
318	092	14	13	1	5	09	6	072	39	33	177	1	075	4	2
319	103	18	13	5	5	13	7	081	43	38	172	0	083	3	3
320	095	18	14	4	6	12	4	054	31	23	164	1	075	4	2
321	093	15	12	3	5	10	6	058	29	29	160	0	081	4	3
322	100	14	10	4	4	10	5	075	42	33	170	0	072	3	2
323	099	12	10	2	4	08	7	072	36	36	168	1	061	3	2
324	102	15	10	5	5	10	5	072	40	32	163	1	082	3	3
325	099	18	13	5	6	12	7	088	46	42	169	0	087	3	3
326	089	09	07	2	2	07	5	062	22	40	167	1	065	4	2
327	090	10	07	3	5	05	3	044	22	22	192	1	073	4	2
328	106	13	10	3	3	10	7	089	52	37	167	0	081	2	3
329	123	23	17	6	6	17	9	103	53	50	170	1	089	1	3
330	101	11	08	3	3	08	4	064	29	35	163	1	090	3	3
331	090	11	08	3	6	05	4	049	26	23	175	1	101	4	3
332	101	14	10	4	4	10	6	061	31	30	169	1	074	3	2
333	094	20	14	6	5	15	5	051	25	26	152	1	075	4	2
334	108	19	15	4	6	13	4	084	47	37	163	1	095	2	3
335	114	19	15	4	5	14	5	090	46	44	167	0	067	2	2
336	120	19	14	5	6	13	9	105	59	46	165	1	098	1	3
337	100	14	11	3	4	10	6	065	38	27	169	0	110	3	4
338	092	14	11	3	4	10	5	065	36	29	173	1	081	4	3
339	120	20	16	4	5	15	9	095	49	46	167	1	063	1	2
340	104	16	12	4	5	11	6	082	45	37	172	1	098	3	3
341	118	18	15	3	6	12	9	101	53	48	169	0	063	1	2
342	093	12	08	4	4	08	4	056	23	33	174	1	078	4	2
343	104	17	14	3	6	11	7	090	48	42	168	0	066	3	2
344	088	19	13	6	4	15	5	054	21	33	174	1	110	4	4
345	088	12	08	4	5	07	4	060	32	28	172	0	083	4	3
501	078	08	06	2	5	03	1	037	20	17	187	0	066	5	2
502	121	21	15	6	5	16	4	078	45	33	156	1	071	1	2
503	123	22	16	6	6	16	8	095	52	43	158	0	069	1	2
504	107	24	18	6	6	18	7	083	44	39	174	1	103	2	4
505	108	10	06	4	5	05	1	062	40	22	180	0	071	2	2
506	093	19	14	5	5	14	3	064	33	31	171	1	110	4	4

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
507	103	23	17	6	5	18	7	086	45	41	168	1	090	3	3
508	115	21	16	5	6	15	5	087	49	38	167	0	073	2	2
509	109	20	14	6	3	17	5	081	35	46	164	0	066	2	2
510	084	11	08	3	5	06	3	054	27	27	177	0	123	5	4
511	092	07	04	3	1	06	3	052	27	25	195	0	071	4	2
512	100	12	09	3	4	08	5	064	27	37	171	0	089	3	3
513	093	14	08	6	4	10	2	069	42	27	190	0	062	4	2
514	103	14	11	3	5	09	6	066	31	35	164	0	084	3	3
515	123	23	17	6	6	17	7	107	59	48	468	0	073	1	2
516	100	13	10	3	5	08	7	077	38	39	170	0	096	3	3
517	099	08	06	2	4	04	2	062	46	16	177	1	115	3	4
518	109	10	13	6	5	14	5	073	45	28	163	1	099	2	3
519	114	22	16	6	6	16	6	098	55	43	166	0	058	2	2
520	109	11	07	4	4	07	1	070	38	32	168	0	085	2	3
521	115	18	13	5	6	12	4	088	53	35	180	1	122	2	4
522	124	18	12	6	5	13	7	090	52	38	166	0	060	1	2
523	103	20	15	5	5	15	4	098	38	30	178	1	131	3	4
524	099	07	05	2	2	05	1	061	34	27	172	1	071	3	2
525	094	17	12	5	4	13	4	057	28	29	163	1	056	4	2
526	115	23	16	7	6	17	8	082	45	37	169	1	120	2	4
527	114	17	11	6	5	12	7	094	54	40	164	0	079	2	2
528	110	16	11	5	5	11	5	067	30	37	173	1	089	2	3
529	095	10	06	4	4	06	3	063	38	25	190	1	115	4	4
530	088	14	10	4	4	10	3	071	44	27	178	1	087	4	3
531	100	10	06	4	4	06	4	063	37	26	168	0	081	3	3
532	095	10	07	3	4	06	1	048	36	12	181	1	102	4	3
533	110	12	10	2	2	10	1	059	32	27	167	1	100	2	3
534	096	09	05	4	4	05	2	056	25	31	166	0	080	3	3
535	100	14	10	4	5	09	7	076	38	38	166	0	115	3	4
536	116	21	15	6	6	15	4	086	46	40	171	1	116	1	4
537	098	10	07	3	4	09	1	057	31	26	171	1	062	3	2
538	096	06	05	1	4	02	2	050	29	21	177	0	073	3	2
539	104	06	03	3	4	02	2	066	45	21	174	1	060	3	2
540	113	10	06	4	3	07	4	059	34	25	169	0	080	2	3
541	092	07	06	1	3	04	3	054	33	21	183	0	091	4	3
542	110	17	13	4	5	12	5	071	36	35	171	0	069	2	2
543	107	17	13	4	6	11	7	069	33	36	160	0	089	2	3
544	099	15	10	5	4	11	1	052	28	24	168	1	066	4	2
545	118	22	17	5	6	16	5	091	53	38	171	1	124	1	4
546	094	09	07	2	5	04	2	046	30	16	181	0	115	4	4
547	098	13	06	7	5	08	3	069	38	31	172	1	089	3	3
548	106	14	08	6	5	09	7	084	40	44	170	0	089	2	3
549	118	23	18	5	6	17	9	102	54	48	167	1	096	1	3
550	093	12	09	3	6	06	3	058	38	20	166	1	094	4	3

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
551	110	20	17	3	5	15	6	085	41	44	166	1	085	2	3
552	091	12	10	2	2	10	1	061	44	17	192	0	129	4	4
553	115	14	10	4	5	09	5	092	54	38	171	1	071	2	2
554	117	14	10	4	6	08	3	069	40	29	165	1	085	1	3
555	101	15	11	4	4	11	4	070	35	35	174	1	089	3	3
556	109	09	08	1	4	05	4	075	46	29	166	0	158	2	4
557	102	14	10	4	4	10	6	082	41	41	170	1	104	3	4
558	114	19	14	5	5	14	6	094	52	42	172	0	069	2	2
601	075	07	06	1	3	04	3	049	29	20	194	1	074	5	2
602	093	15	10	5	5	10	4	052	19	33	177	0	089	4	3
603	091	17	14	3	5	12	6	075	33	42	166	1	070	4	2
604	094	18	13	5	5	13	5	057	21	36	171	1	079	4	2
605	114	19	14	5	5	14	5	090	44	46	166	1	034	2	2
606	121	19	14	5	6	13	8	089	43	46	173	0	075	1	2
607	089	10	08	2	5	05	3	043	19	24	169	1	060	4	2
608	091	17	14	3	5	12	4	055	27	28	179	1	061	4	2
609	092	06	04	2	2	04	3	046	21	25	168	1	062	4	2
610	114	20	15	5	5	15	7	089	48	41	169	0	066	2	2
611	097	14	10	4	3	11	5	069	31	38	168	0	051	3	2
612	091	06	04	2	1	05	2	063	33	30	160	0	063	4	2
613	107	11	09	2	3	08	4	070	34	36	168	0	054	2	2
614	107	17	13	4	5	12	7	074	35	39	168	0	054	2	2
615	106	11	08	3	4	07	3	064	31	33	167	1	082	2	3
616	078	13	09	4	5	08	4	049	13	36	183	1	055	5	2
617	112	25	20	5	6	19	7	089	48	41	168	1	055	2	2
618	120	22	16	6	6	16	8	095	52	43	163	0	074	1	2
619	096	04	03	1	2	02	1	042	20	22	169	1	080	3	3
620	112	22	18	4	5	17	8	100	53	47	173	1	071	2	2
621	114	16	13	3	5	11	5	084	42	42	173	1	062	2	2
622	101	12	09	3	4	08	4	064	36	28	162	0	059	3	2
623	119	18	14	4	5	13	7	096	53	43	167	0	079	1	2
624	095	12	08	4	3	09	6	054	28	26	166	0	055	4	2
625	107	19	15	4	5	14	7	084	44	40	173	1	077	2	2
626	109	22	17	5	6	16	6	080	39	41	170	1	077	2	2
627	084	03	03	0	2	01	1	029	14	15	186	1	074	5	2
628	098	08	06	2	5	03	3	029	13	16	164	0	062	3	2
629	086	12	10	2	5	07	4	040	16	24	173	1	083	4	3
630	107	22	17	5	6	16	7	084	42	42	173	0	076	2	2
631	114	07	06	1	3	04	4	070	44	26	170	1	071	2	2
632	107	15	12	3	4	11	3	063	28	35	162	0	070	2	2
633	097	09	07	2	4	05	4	049	18	31	168	0	047	3	2
634	076	01	00	1	1	00	1	030	17	13	181	1	061	5	2
635	088	13	11	2	4	09	4	041	25	16	170	1	058	4	2
636	110	18	14	4	6	12	6	079	44	35	162	0	077	2	2
637	099	14	11	3	5	09	5	057	27	30	163	0	070	3	2

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
701	122	21	17	4	6	15	8	098	52	46	172	1	063	1	2
702	096	13	08	5	4	09	6	052	14	38	168	0	067	3	2
703	113	08	07	1	3	05	3	078	55	23	170	1	073	2	2
704	101	04	02	2	2	02	5	077	46	31	174	0	081	3	3
705	090	09	05	4	3	06	4	067	34	33	167	1	073	4	2
706	107	21	17	4	6	15	5	080	43	37	169	1	088	2	3
707	085	08	04	4	4	04	3	041	22	19	166	1	086	5	3
708	116	15	11	4	5	10	7	083	50	33	166	1	086	1	3
709	097	11	09	2	5	06	5	057	29	28	165	1	080	3	3
710	111	23	18	5	6	17	8	090	48	42	175	0	075	2	2
711	096	09	07	2	4	05	4	058	28	30	184	1	060	3	2
712	119	23	18	5	6	17	7	081	40	41	162	1	080	1	3
713	133	16	13	3	6	10	5	093	60	33	152	0	073	1	2
714	079	06	04	2	3	03	3	034	18	16	189	1	090	5	3
715	112	15	11	4	5	10	7	073	34	39	164	1	072	2	2
716	082	08	06	2	4	04	4	053	20	33	177	1	090	5	3
717	102	10	07	3	5	05	4	059	34	25	166	0	075	3	2
718	106	14	11	3	4	10	5	071	37	34	164	0	110	2	4
719	104	07	06	1	3	04	4	073	50	23	173	1	075	3	2
720	119	21	15	6	6	15	6	076	44	32	168	1	082	1	3
721	098	14	10	4	5	09	7	051	19	32	165	0	086	3	3
722	103	16	14	2	4	12	6	095	54	41	175	1	062	3	2
723	102	11	09	2	4	07	4	067	43	24	170	0	072	3	2
724	115	14	10	4	4	10	5	089	55	34	169	1	060	2	2
725	107	18	14	4	5	13	6	078	38	40	172	1	093	2	3
726	093	02	02	0	0	02	4	040	26	14	172	0	081	4	3
727	114	19	14	5	3	16	7	081	44	37	172	0	071	2	2
728	109	11	08	3	3	08	4	065	31	34	169	1	060	2	2
729	118	19	15	4	6	13	8	105	56	49	161	1	060	1	2
730	085	05	05	0	3	02	1	034	19	15	175	1	087	5	3
731	106	15	11	4	4	11	4	067	32	35	170	1	068	2	2
732	127	19	16	3	5	14	9	100	55	45	174	1	064	1	2
733	116	20	16	4	6	14	8	101	55	46	174	1	091	1	3
734	098	12	09	3	4	08	5	059	27	32	168	0	060	3	2
735	109	14	10	4	5	09	6	080	42	38	169	1	063	2	2
736	107	14	09	5	4	10	7	074	41	33	169	0	082	2	3
737	102	12	10	2	4	08	2	067	45	22	184	0	095	3	3
738	111	11	08	3	5	06	5	079	44	35	162	0	095	2	3
739	099	08	06	2	4	04	4	050	30	20	164	0	103	3	4
740	103	06	03	3	3	03	5	048	21	27	164	0	094	3	3
741	118	17	12	5	6	11	8	097	52	45	167	1	077	1	2
742	115	20	15	5	6	14	7	093	47	46	168	1	068	2	2
743	124	19	14	5	5	14	6	098	54	44	166	0	091	1	3
744	106	12	09	3	3	09	5	072	31	41	170	1	060	2	2

APPENDIX E

CORRELATION COEFICIENTS BETWEEN THE FOUR VARIABLES, OTIS, PS8, M9, and SCAT

Variables	OTIS	PS8	M9	SCAT
OTIS	-	0.638 (13.478)	0.618 (12.782)	0.826 (23.811)
PS8	-	-	0.726 (17.177)	0.754 (18.675)
M9	-	-	-	0.722 (16.977)
SCAT	-	-	-	-

(t-ratio) all are significant $t_{.01} = (2.576)$ df = 264